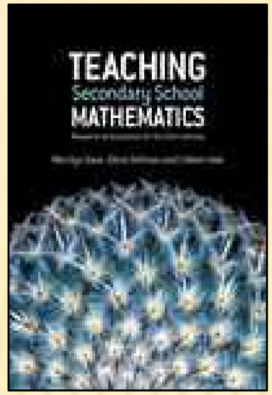
TEXT: TEACHING SECONDARY SCHOOL MATHEMATICS



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TEXT OUTLINE

- PART 1 Introduction
- PART 2 Math Pedagogy, Curriculum, Assessment
- PART 3 Teaching and Learning Math Content
- PART 4 Equity and Diversity in Math Education
- PART 5 Professional and Community Engagement

CHAPTER 2 OUTLINE

- (1) Understanding in mathematics

- (2) Learning theories
- (3) Mathematical thinking
- (4) Role of the teacher in creating a culture of mathematical inquiry

(1) HOW DO YOU KNOW WHEN YOU UNDERSTAND SOMETHING IN MATH?

- Students associated understanding with…

✓ GETTING CORRECT ANSWER	234	71%
✓ Feel confident or interested	35	11%
✓ Knowing why, making sense	52	16%
✓ Being able to apply elsewhere	27	8%
✓ EXPLAIN TO SOMEONE ELSE	24	7%

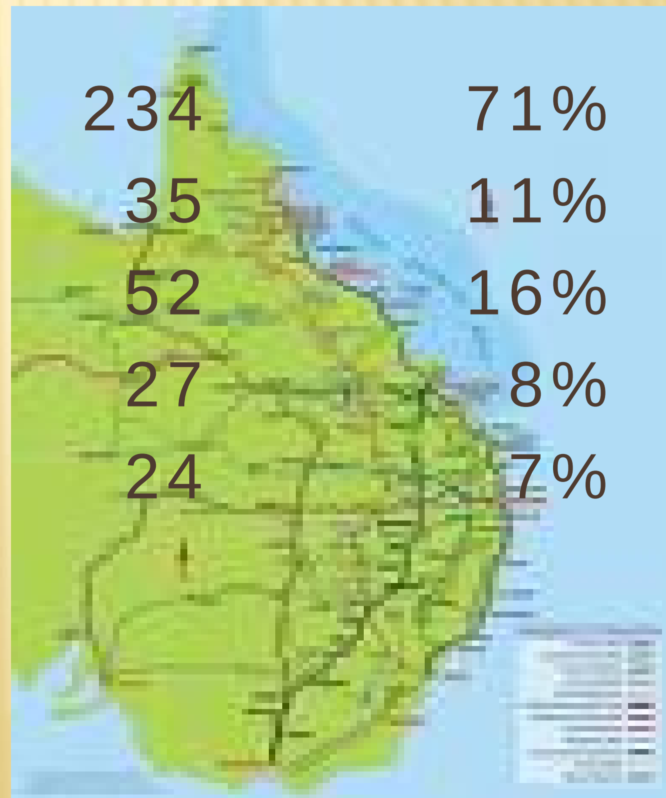


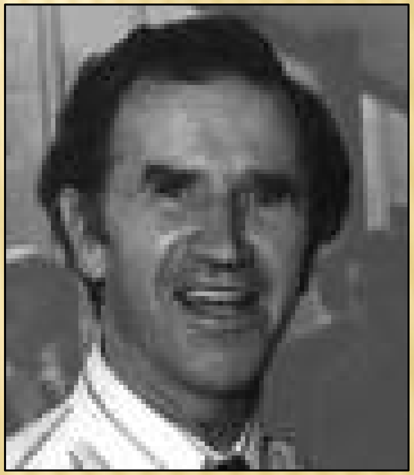
Table 2.1 page 23

(1) HOW DO YOU KNOW WHEN YOU UNDERSTAND SOMETHING IN MATH?

- 2 different kinds of MATHEMATICAL UNDERSTANDING :

- Instrumental Understanding

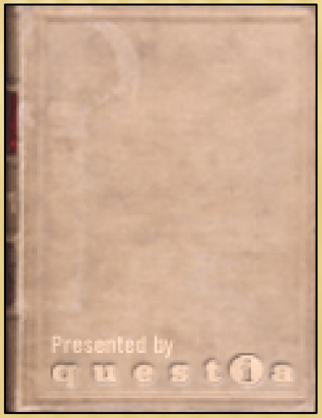
  -knowing WHAT to do to solve a problem



Richard R. Skemp

- Relational Understanding

  -knowing WHAT to do and WHY you're doing it



(1) HOW DO YOU KNOW WHEN YOU UNDERSTAND SOMETHING IN MATH?

- 4 different kinds of MATHEMATICAL UNDERSTANDING

(1) knowing ***that*** (stating something);
EX1: the sum of interior angles of triangle is ....

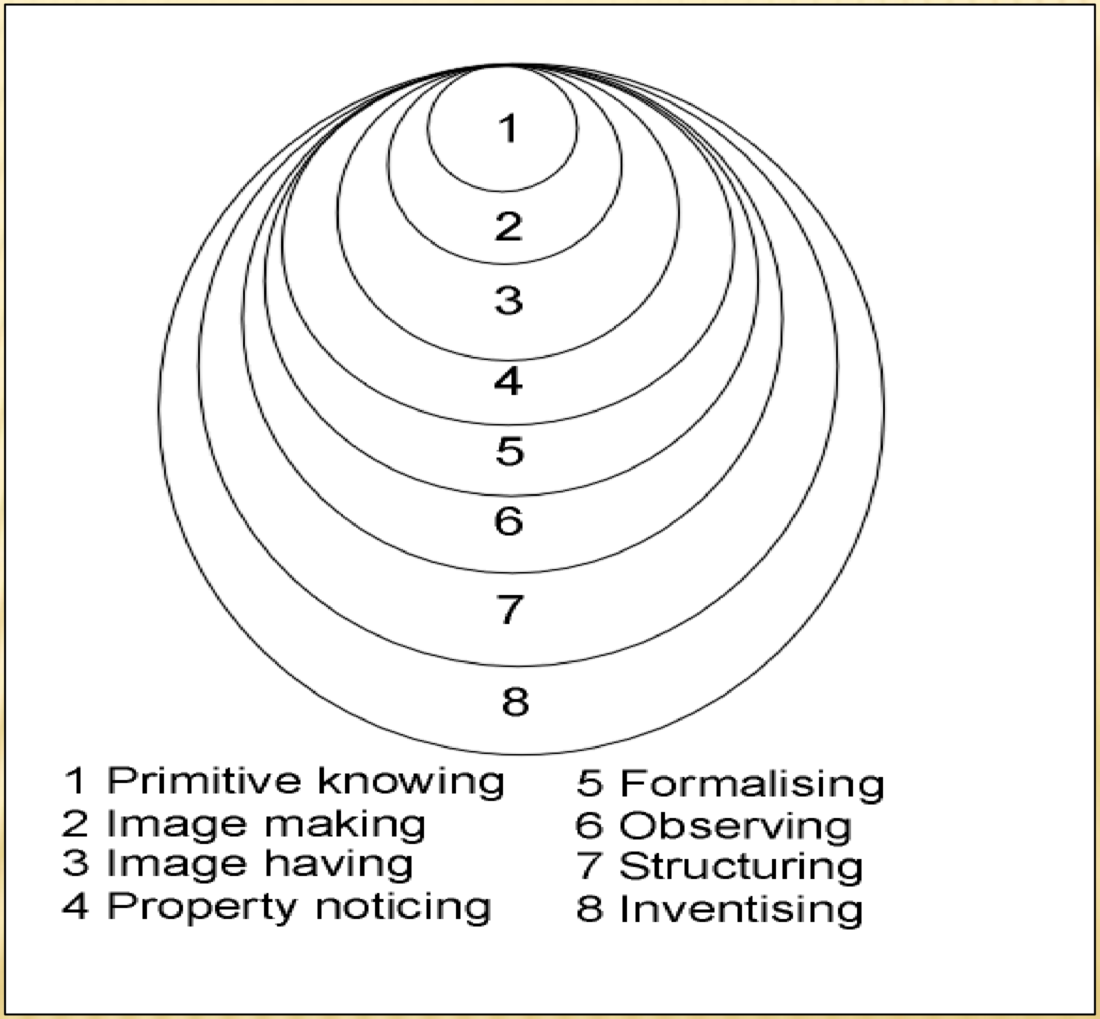
(2) knowing ***how*** (doing something); *EX2: measure angles*

(3) knowing ***why*** (explaining or proving something); EX1

(4) knowing ***to*** (as in recognizing the opportunity when to use a mathematical idea in working on a problem)
EX3: the sum of interior angles of a n-polygon is .....

John Mason

(1) HOW DO YOU KNOW WHEN YOU UNDERSTAND SOMETHING IN MATH?



Pirie-Kieren theory of mathematical understanding

(1) HOW DO YOU KNOW WHEN YOU UNDERSTAND SOMETHING IN MATH?

(1) PRIMITIVE KNOWING (where I am): starting point

(2) IMAGE MAKING: learners use previous knowledge

(3) IMAGE HAVING: mental construction of problem without moving physical objects

(4) PROPERTY NOTICING: combine aspects of several properties

(5) FORMALIZING: learner abstracts a quality from the previous image

(6) OBSERVING: reflecting on and coordinating theorems

(7) STRUCTURING: develop a theory about the problem

(8) INVENTISING: breaking away from existing understanding and creating new ideas

(1) HOW DO YOU KNOW WHEN YOU UNDERSTAND SOMETHING IN MATH?

- School mathematics vs. Inquiry mathematics

  (Procedure) vs. (Understanding)

- Students often make mistakes when they follow procedure without basic understanding

- “errors are based on systematic *rules* which are usually distortions of sound procedures”

  (T. Perso, 1992)

(1) HOW DO YOU KNOW WHEN YOU UNDERSTAND SOMETHING IN MATH?

School mathematics vs. Inquiry mathematics
(Procedure) vs. (Understanding)

$$\begin{array}{lcl} 2X + Y & = & 10 \\ X - Y & = & 2 \\ \hline X & = & 8 \end{array}$$

(incorrect)

Student subtracted the equations, should have added them

$$\begin{array}{lcl} 2X + Y & = & 10 \\ X - Y & = & 2 \\ \hline 3X & = & 12 \end{array}$$

(X=4, Y=2)

Conclusion: students shouldn’t aim to reproduce procedures without understanding why they work. It leads to flawed logic.

(1) HOW DO YOU KNOW WHEN YOU UNDERSTAND SOMETHING IN MATH?

Jo Boaler

Amber Hill	Phoenix Park
School mathematics culture	Inquiry mathematics culture
Instrumental Understanding	Relational Understanding
Traditional teaching methods	Progressive, open-ended
Not helped by formal tests	Flexible and adaptive thinkers
Knowing what to, how to	Knowing when to, why to

(2) TWO THEORIES OF LEARNING (CONSTRUCTIVISM AND SOCIO-CULTURAL PERSPECTIVES)

CONSTRUCTIVISM

- Gives priority to **individual construction of** mathematical **understanding**
- Sees **social interaction as a source of cognitive conflict** that brings about learning through reorganization of mental structures
- **Students actively construct knowledge by connecting prior information with new information gained through interactions with the world**

Jean Piaget

(2) TWO THEORIES OF LEARNING (CONSTRUCTIVISM AND SOCIO-CULTURAL PERSPECTIVES)

- Vygotsky's socio-cultural perspective of learning says that **individual cognition has its origins in social interaction** (importance of language, culture, traditions)

- Memory and reasoning appear first between people as social processes, then within an individual as internal mental processes

- ZPD: "zone of proximal development"

- **Scaffolding**
- **Collaborative group work**

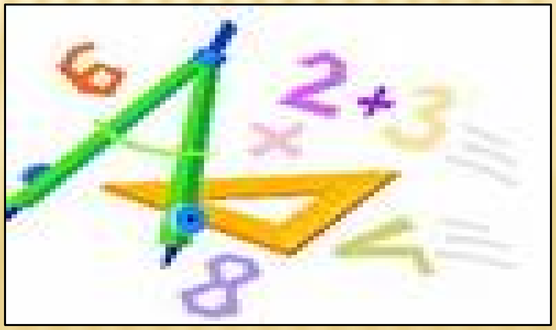
Lev Vygotsky

(2) TWO THEORIES OF LEARNING (CONSTRUCTIVISM AND SOCIO-CULTURAL PERSPECTIVES)

SOCIO-CULTURAL PERSPECTIVES

- Table 2.4 (page 39 of text) Teacher **scaffolding questions** during a problem-solving exercise

Getting started	While students working	After students finished
What are important ideas?	Tell me what you are doing	Have you answered question?
Can you rephrase problem?	Why did you think of that?	Have you considered all cases?
What are you asked to find?	Why are you doing this?	Have you checked your answer?
What information is given?	What will you do with the result?	Can you explain your answer?
Seen problem like this before?	Why is that idea better than this?	Another way to solve problem?



(2) TWO THEORIES OF LEARNING (CONSTRUCTIVISM AND SOCIO-CULTURAL PERSPECTIVES)

A classroom scenario:

- First attempt was to offer simple examples to students (unsuccessful)
- Second attempt was to try a **practical activity** where students worked out for themselves the properties of equilateral and isosceles triangles using ruler and compass
- Successful approach because students:
  - -learned by doing
  - -developed increased expectations
  - -wanted to understand
  - -explained to each other
  - -asked questions
  - -co-operated

(2)Brunner's stages of representation in learning

- enactive
- iconic
- symbolic

Use of **manipulatives**, hands-on approach in teaching math prior to formalization and theoretical presentation.

(2) Van Hiele model in learning geometry

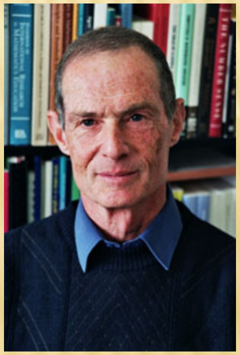
- Visualization (shape recognition, identification)
- Analysis (attributes and properties of shape)
- Informal deduction (compare and classify shapes, simple proofs)
- Deduction (proofs of theorems from postulates)
- Rigor (abstract, proof-oriented reasoning)

Two persons who reason at different levels cannot understand each other.

(3) MATHEMATICAL THINKING

3 categories of student thinking

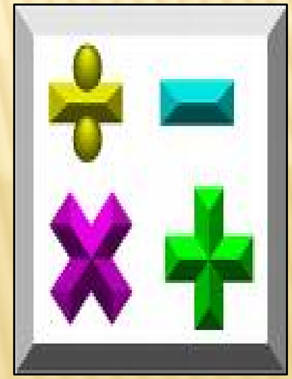
(a) recognizing	realizing that a known mathematical procedure applies in a new situation
(b) building with	using several previously known mathematical procedures to solve unfamiliar problems
(c) constructing	selecting previously known strategies and ideas to integrate when solving a challenging problem



Tom Dreyfus

Gaye Williams

(3) MATHEMATICAL THINKING

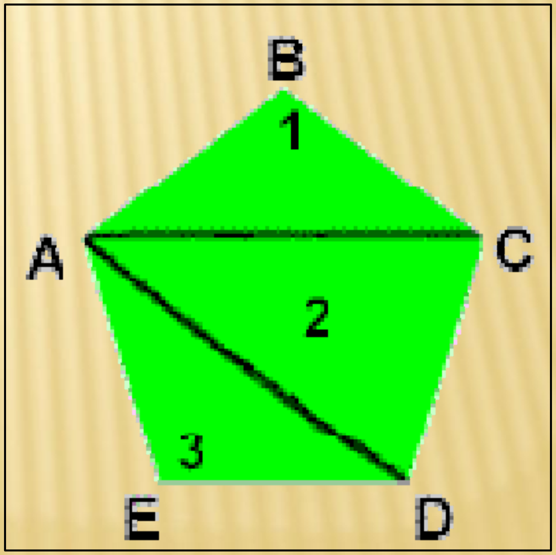


Complexity of thinking	Examples of thinking
Recognizing	Recognizing that a problem involving circles could involve certain formulas
Building with	Searching for patterns, alternative solutions
Constructing	Continually checking for inconsistencies

(3) MATHEMATICAL THINKING

Example involving angles in polygons

- Students were given a page containing several polygons having 3 to 10 sides
- Objective: join pieces together to find the sum of all angles for each polygon
- This involves the 3 levels of thinking :
  - -<u>recognizing</u> the sum of angles for a triangle is 180
  - -<u>building</u> a figure out of the pieces
  - -<u>constructing</u> a table and a theory



pentagon

(3) MATHEMATICAL THINKING

"reasoning" and "problem solving"

Table 2.3 (Representation of process aspects of mathematics in curriculum materials)

National Profile (Australia)	Investigating, conjecturing, applying, verifying (/reasoning) Using problem solving strategies Using mathematical language Working in context
NCTM Principles and Standards (USA)	**Problem solving** **Reasoning and proof** **Communication** **Connections** Representation
National Curriculum (UK)	Problem solving Reasoning Communicating

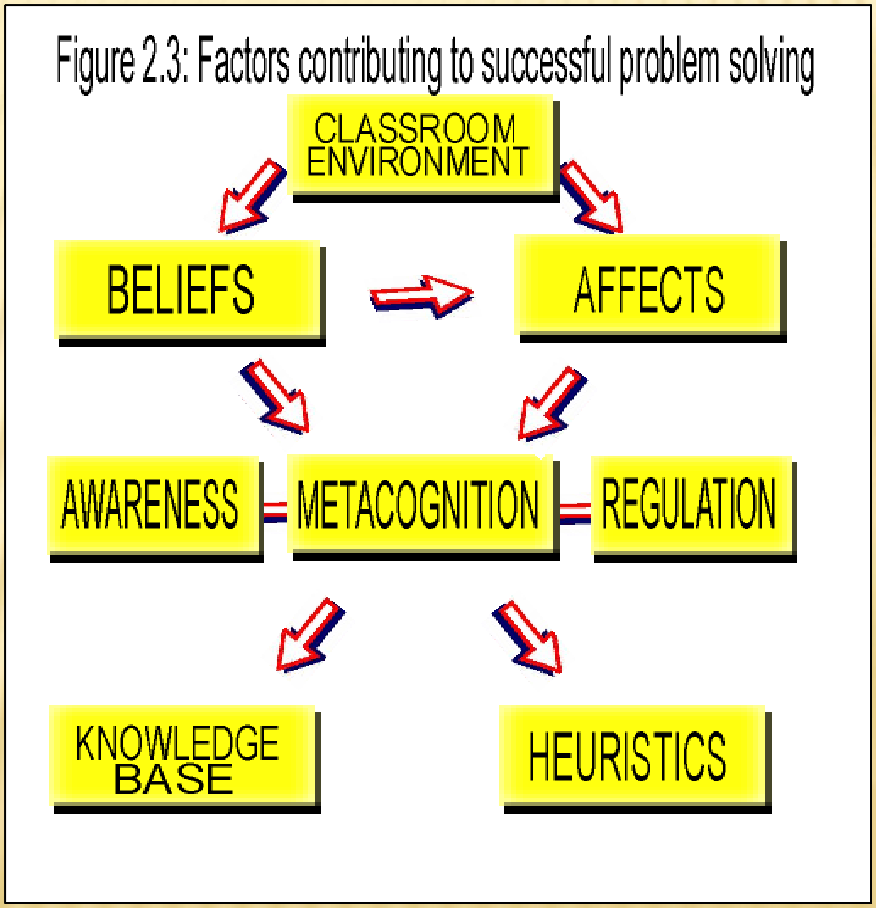
(3) MATHEMATICAL THINKING

Mathematical reasoning

- Involves making, evaluating and investigating **conjectures and guesses**
- Also involves **explanation and justification** for steps that a student takes

(3) MATHEMATICAL THINKING

- Mathematical problem solving
- Figure 2.3 (Factors contributing to successful problem solving)



(3) G. Polya "How to solve it ?"

Steps in problem solving:

- *understand the problem*.
- *make a plan*.
- *carry out the plan*.
- *look back* on your work.

Check for consistency. Is there a better/another solution?

(4) ROLE OF THE TEACHER IN CREATING A CULTURE OF MATHEMATICAL INQUIRY

- Teachers should try to “create a classroom community of inquiry”.
- **Key elements of the teacher’s role are:**

(1) modeling mathematical thinking

(2) asking questions that “scaffold” students’ thinking

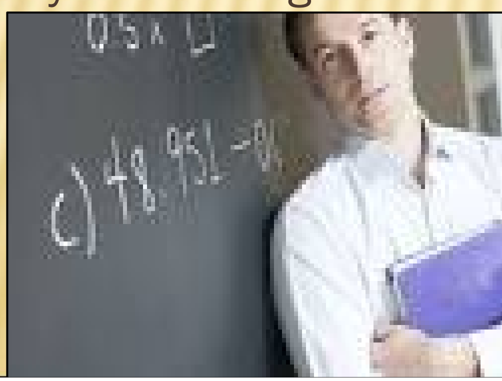
(3) structuring students’ social interactions

(4) connecting students’ developing ideas to mathematical language and symbolism

(4) ROLE OF THE TEACHER IN CREATING A CULTURE OF MATHEMATICAL INQUIRY

Characteristics of a classroom community of mathematical inquiry:
(Table 2.6 page 44)

Dimension	Change
Questioning	Shift from teacher to student as questioner
Explaining mathematical thinking	Teacher asks more probing questions
Source of mathematical Ideas	Shift from teacher as source of all ideas; use students' errors as a learning tool
Responsibility for learning	Students increasingly take this responsibility; self-regulation and self-evaluation



REVIEW OF CHAPTER 2

(1) How do you know when you understand something in math?

(2) What do we know from learning theories?

(3) What is mathematical thinking?

(4) What is the role of the teacher in creating a culture of mathematical inquiry.


THANK YOU

ON THE LIGHTER SIDE