- 117. A city commission has proposed two tax bills. The first bill requires that a homeowner pay \$1800 plus 3% of the assessed home value in taxes. The second bill requires taxes of \$200 plus 8% of the assessed home value. What price range of home assessment would make the first bill a better deal?
- 118. The percentage, p, of defective products manufactured by a company is given by $|p 0.3\%| \le 0.2\%$. If 100,000 products are manufactured and the company offers a \$5 refund for each defective product, describe the company's cost for refunds.

CHAPTER SUMMARY, REVIEW, AND TEST

Summary: Basic Formulas

Definition of Absolute Value

$$|x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$$

Distance between Points a and b on a Number Line

$$|a-b|$$
 or $|b-a|$

Properties of Algebra

Commutative	a+b=b+a, $ab=ba$
Associative	(a+b)+c=a+(b+c)
90	(ab)c = a(bc)
Distributive Identity	a(b+c)=ab+ac
	$a+0=a a\cdot 1=a$
Inverse	$a + (-a) = 0$ $a \cdot \frac{1}{a} = 1, a \neq 0$

Properties of Exponents

$$b^{-n} = \frac{1}{b^n}, \quad b^0 = 1, \quad b^m \cdot b^n = b^{m+n},$$

$$(b^m)^n = b^{mn}, \quad \frac{b^m}{b^n} = b^{m-n}, \quad (ab)^n = a^n b^n, \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Product and Quotient Rules for nth Root

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab} \qquad \sqrt[n]{\frac{a}{b}} = \sqrt[n]{\frac{a}{b}}$$

Rational Exponents

Rational Exponents
$$a^{1/n} = \sqrt[n]{a}, \quad a^{-1/n} = \frac{1}{a^{1/n}} = \frac{1}{\sqrt[n]{a}},$$

$$a^{m/n} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}, \quad a^{-m/n} = \frac{1}{a^{m/n}}$$

Special Products

$$(A + B)(A - B) = A^{2} - B^{2}$$

$$(A + B)^{2} = A^{2} + 2AB + B^{2}$$

$$(A - B)^{2} = A^{2} - 2AB + B^{2}$$

$$(A + B)^{3} = A^{3} + 3A^{2}B + 3AB^{2} + B^{3}$$

$$(A - B)^{3} = A^{3} - 3A^{2}B + 3AB^{2} - B^{3}$$

Factoring Formulas

$$A^{2} - B^{2} = (A + B)(A - B)$$

$$A^{2} + 2AB + B^{2} = (A + B)^{2}$$

$$A^{2} - 2AB + B^{2} = (A - B)^{2}$$

$$A^{3} + B^{3} = (A + B)(A^{2} - AB + B^{2})$$

$$A^{3} - B^{3} = (A - B)(A^{2} + AB + B^{2})$$

The Quadratic Formula

All quadratic equations

$$ax^2 + bx + c = 0, a \neq 0$$

can be solved by the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

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Review Exercises

You can use these review exercises, like the review exercises at the end of each chapter, to test your understanding of the chapter's topics. However, you can also use these exercises as a prerequisite test to check your mastery of the fundamental algebra skills needed in this book.

P.1

1. Consider the set:

$$\{-17, -\frac{9}{13}, 0, 0.75, \sqrt{2}, \pi, \sqrt{81}\}.$$

List all numbers from the set that are a. natural numbers, b. whole numbers, c. integers, d. rational numbers, e. irrational numbers.

In Exercises 2-4, rewrite each expression without absolute value bars.

3.
$$|\sqrt{2}-1|$$

4.
$$|3 - \sqrt{17}|$$

5. Express the distance between the numbers -17 and 4 using absolute value. Then evaluate the absolute value.

In Exercises 6-7, evaluate each algebraic expression for the ziven value of the variable.

6.
$$\frac{5}{9}(F-32)$$
; $F=68$ 7. $\frac{8(x+5)}{3x+8}$, $x=2$

7.
$$\frac{8(x+5)}{3x+8}$$
, $x=2$

In Exercises 8-13, state the name of the property illustrated.

8.
$$3 + 17 = 17 + 3$$

9.
$$(6 \cdot 3) \cdot 9 = 6 \cdot (3 \cdot 9)$$

$$(0. \sqrt{3}(\sqrt{5} + \sqrt{3}) = \sqrt{15} + 3)$$

11.
$$(6 \cdot 9) \cdot 2 = 2 \cdot (6 \cdot 9)$$

2.
$$\sqrt{3}(\sqrt{5} + \sqrt{3}) = (\sqrt{5} + \sqrt{3})\sqrt{3}$$

3.
$$(3 \cdot 7) + (4 \cdot 7) = (4 \cdot 7) + (3 \cdot 7)$$

n Exercises 14-15, simplify each algebraic expression.

4.
$$3(7x-5y)-2(4y-x+1)$$

5.
$$\frac{1}{5}(5x) + [(3y) + (-3y)] - (-x)$$

2.2

6.
$$(-3)^3(-2)^2$$

17.
$$2^{-4} + 4^{-1}$$

19.
$$\frac{3^3}{3^6}$$

implify each exponential expression in Exercises 20-23.

valuate each exponential expression in Exercises 16-19.

0.
$$(-2x^4y^3)^3$$

21.
$$(-5x^3y^2)(-2x^{-11}y^{-2})$$

23. $\frac{7x^5y^6}{28x^{15}y^{-2}}$

2.
$$(2x^3)^{-4}$$

23.
$$\frac{7x^3y^6}{28x^{15}y^{-1}}$$

In Exercises 24-25, write each number in decimal notation.

24.
$$3.74 \times 10^4$$

25.
$$7.45 \times 10^{-5}$$

In Exercises 26-27, write each number in scientific notation.

27. 0.00725

In Exercises 28-29, perform the indicated operation and write the answer in decimal notation.

28.
$$(3 \times 10^3)(1.3 \times 10^2)$$

29.
$$\frac{6.9 \times 10^3}{3 \times 10^5}$$

30. If you earned \$1 million per year (\$106), how long would it take to accumulate \$1 billion (\$109)?

31. If the population of the United States is 2.8×10^8 and each person spends about \$150 per year going to the movies (or renting movies), express the total annual spending on movies in scientific notation.

P.3

Use the product rule to simplify the expressions in Exercises 32-35. In Exercises 34-35, assume that variables represent nonnegative real numbers.

32.
$$\sqrt{300}$$

3.
$$\sqrt{12x^2}$$

34.
$$\sqrt{10x} \cdot \sqrt{2x}$$

33.
$$\sqrt{12x^2}$$
 35. $\sqrt{r^3}$

Use the quotient rule to simplify the expressions in Exercises

36.
$$\sqrt{\frac{121}{4}}$$

37.
$$\frac{\sqrt{96x^3}}{\sqrt{2x}}$$
 (Assume that $x > 0$.)

In Exercises 38-40, add or subtract terms whenever possible.

38.
$$7\sqrt{5} + 13\sqrt{5}$$

39.
$$2\sqrt{50} + 3\sqrt{8}$$

40.
$$4\sqrt{72} - 2\sqrt{48}$$

In Exercises 41-44, rationalize the denominator.

41.
$$\frac{30}{\sqrt{5}}$$

42.
$$\frac{\sqrt{2}}{\sqrt{3}}$$

43.
$$\frac{5}{6+\sqrt{3}}$$

44.
$$\frac{14}{\sqrt{7}-\sqrt{5}}$$

Evaluate each expression in Exercise 45-48 or indicate that the root is not a real number.

45.
$$\sqrt[3]{125}$$

46.
$$\sqrt[5]{-32}$$

48.
$$\sqrt[4]{(-5)^4}$$

Simplify the radical expressions in Exercises 49-53.

49.
$$\sqrt[3]{81}$$

50.
$$\sqrt[3]{y^5}$$

51.
$$\sqrt[4]{8} \cdot \sqrt[4]{10}$$

52.
$$4\sqrt[3]{16} + 5\sqrt[3]{2}$$

53.
$$\frac{\sqrt[4]{32x^5}}{\sqrt[4]{16x}}$$
 (Assume that $x > 0$.)

In Exercises 54-59, evaluate each expression.

In Exercises 60-62, simplify using properties of exponents.

60.
$$(5x^{2/3})(4x^{1/4})$$

61.
$$\frac{15x^{3/4}}{5x^{1/2}}$$

62.
$$(125x^6)^{2/3}$$

63. Simplify by reducing the index of the radical:
$$\sqrt[6]{y^3}$$
.

P.4

In Exercises 64-65, perform the indicated operations. Write the resulting polynomial in standard form and indicate its degree.

64.
$$(-6x^3 + 7x^2 - 9x + 3) + (14x^3 + 3x^2 - 11x - 7)$$

65.
$$(13x^4 - 8x^3 + 2x^2) - (5x^4 - 3x^3 + 2x^2 - 6)$$

In Exercises 66-72, find each product.

66.
$$(3x-2)(4x^2+3x-5)$$
 67. $(3x-5)(2x+1)$

67.
$$(3x-5)(2x+1)$$

68.
$$(4x + 5)(4x - 5)$$
 69. $(2x + 5)^2$

69.
$$(2x + 5)^2$$

70.
$$(3x-4)^2$$

71.
$$(2x+1)^3$$

72.
$$(5x-2)^3$$

In Exercises 73-79, find each product.

73.
$$(x + 7y)(3x - 5y)$$

74.
$$(3x - 5y)^2$$

75.
$$(3x^2 + 2y)^2$$

76.
$$(7x + 4y)(7x - 4y)$$

77.
$$(a-b)(a^2+ab+b^2)$$

78.
$$[5y - (2x + 1)][5y + (2x + 1)]$$

79.
$$(x + 2y + 4)^2$$

In Exercises 80-96, factor completely, or state that the polynomial is prime.

80.
$$15x^3 + 3x^2$$

81.
$$x^2 - 11x + 28$$

82.
$$15x^2 - x - 2$$

83.
$$64 - x^2$$

84.
$$x^2 + 16$$

85.
$$3x^4 - 9x^3 - 30x^2$$

86.
$$20x^7 - 36x^3$$

87.
$$x^3 - 3x^2 - 9x + 27$$

88.
$$16x^2 - 40x + 25$$

89.
$$x^4 - 16$$

90.
$$y^3 - 8$$

91.
$$x^3 + 64$$

92.
$$3x^4 - 12x^2$$

93.
$$27x^3 - 125$$

94.
$$x^5 - x$$

95.
$$x^3 + 5x^2 - 2x - 10$$

96.
$$x^2 + 18x + 81 - y^2$$

In Exercises 97-99, factor and simplify each algebraic expression.

97.
$$16x^{-3/4} + 32x^{1/4}$$

98.
$$(x^2-4)(x^2+3)^{1/2}-(x^2-4)^2(x^2+3)^{3/2}$$

99.
$$12x^{-1/2} + 6x^{-3/2}$$

P.6

In Exercises 100-102, simplify each rational expression. Also, list all numbers that must be excluded from the domain.

100.
$$\frac{x^3 + 2x}{x + 2}$$

100.
$$\frac{x^3 + 2x^2}{x + 2}$$
 101. $\frac{x^2 + 3x - 18}{x^2 - 36}$

$$102. \ \frac{x^2 + 2x}{x^2 + 4x + 4}$$

In Exercises 103-105, multiply or divide as indicated.

103.
$$\frac{x^2+6x+9}{x^2-4} \cdot \frac{x+3}{x-2}$$
 104. $\frac{6x+2}{x^2-1} \div \frac{3x^2+x}{x-1}$

105.
$$\frac{x^2-5x-24}{x^2-x-12} \div \frac{x^2-10x+16}{x^2+x-6}$$

In Exercises 106-109, add or subtract as indicated.

106.
$$\frac{2x-7}{x^2-9} - \frac{x-10}{x^2-9}$$
 107. $\frac{3x}{x+2} + \frac{x}{x-2}$

107.
$$\frac{3x}{x+2} + \frac{x}{x-2}$$

108.
$$\frac{x}{x^2-9} + \frac{x-1}{x^2-5x+6}$$

109.
$$\frac{4x-1}{2x^2+5x-3} - \frac{x+3}{6x^2+x-2}$$

In Exercises 110-112, simplify each complex rational expression.

110.
$$\frac{3 + \frac{12}{x}}{1 - \frac{16}{x}}$$

110.
$$\frac{3 + \frac{12}{x}}{1 - \frac{16}{x^2}}$$
 111. $\frac{3 - \frac{1}{x+3}}{3 + \frac{1}{x+3}}$

112.
$$\frac{\sqrt{25-x^2}+\frac{x^2}{\sqrt{25-x^2}}}{25-x^2}$$

P.7

In Exercises 113–118, solve and check each linear equation.

113.
$$2x - 5 = 7$$

114.
$$5x + 20 = 3x$$

115.
$$7(x-4) = x+2$$

116.
$$1 - 2(6 - x) = 3x + 2$$

117.
$$2(x-4) + 3(x+5) = 2x-2$$

118.
$$2x - 4(5x + 1) = 3x + 17$$

Exercises 119-123 contain equations with constants in denominators. Solve each equation and check by the method of your choice.

119.
$$\frac{2x}{3} = \frac{x}{6} + 1$$

119.
$$\frac{2x}{3} = \frac{x}{6} + 1$$
 120. $\frac{x}{2} - \frac{1}{10} = \frac{x}{5} + \frac{1}{2}$

121.
$$\frac{2x}{3} = 6 - \frac{x}{4}$$

121.
$$\frac{2x}{3} = 6 - \frac{x}{4}$$
 122. $\frac{x}{4} = 2 + \frac{x-3}{3}$

123.
$$\frac{3x+1}{3} - \frac{13}{2} = \frac{1-x}{4}$$

Exercises 124-127 contain equations with variables in denominators. a. List the value or values representing restriction(s) on the variable. b. Solve the equation.

$$124. \ \frac{9}{4} - \frac{1}{2x} = \frac{4}{x}$$

124.
$$\frac{9}{4} - \frac{1}{2x} = \frac{4}{x}$$
 125. $\frac{7}{x-5} + 2 = \frac{x+2}{x-5}$

126.
$$\frac{1}{x-1} - \frac{1}{x+1} = \frac{2}{x^2-1}$$

$$27. \frac{4}{x+2} + \frac{2}{x-4} = \frac{30}{x^2 - 2x - 8}$$

'n Exercises 128–130, determine whether each equation is an dentity, a conditional equation, or an inconsistent equation.

28.
$$\frac{1}{x+5}=0$$

29.
$$7x + 13 = 4x - 10 + 3x + 23$$

30.
$$7x + 13 = 3x - 10 + 2x + 23$$

31. The percentage, P, of U.S. adults who read the daily newspaper can be described by the formula

$$P = -0.7x + 80$$

where x is the number of years after 1965. In which year will 52% of U.S. adults read the daily newspaper?

1 Exercises 132-134, solve each formula for the specified ariable.

32.
$$V = \frac{1}{3}Bh$$
 for h

32.
$$V = \frac{1}{3}Bh$$
 for h **133.** $F = f(1 - M)$ for M

34.
$$T = gr + gvt$$
 for g

olve the equations containing absolute value in Exercises 35-136.

35.
$$|2x + 1| = 7$$

136.
$$2|x-3|-6=10$$

olve each equation in Exercises 137-138 by factoring.

$$17. \ 2x^2 + 15x = 8$$

138.
$$5x^2 + 20x = 0$$

Solve each equation in Exercises 139-140 by the square root method.

139.
$$2x^2 - 3 = 125$$

140.
$$(3x - 4)^2 = 18$$

In Exercises 141-142, determine the constant that should be added to the binomial so that it becomes a perfect square trinomial. Then write and factor the trinomial.

141.
$$x^2 + 20x$$

142.
$$x^2 - 3x$$

Solve each equation in Exercises 143-144 by completing the square.

143.
$$x^2 - 12x + 27 = 0$$

144.
$$3x^2 - 12x + 11 = 0$$

Solve each equation in Exercises 145-146 using the quadratic formula.

145.
$$x^2 = 2x + 4$$

146.
$$2x^2 = 3 - 4x$$

Compute the discriminant of each equation in Exercises 147-148. What does the discriminant indicate about the number and type of solutions?

147.
$$x^2 - 4x + 13 = 0$$

148.
$$9x^2 = 2 - 3x$$

Solve each equation in Exercises 149-153 by the method of your choice.

149.
$$2x^2 - 11x + 5 = 0$$

150.
$$(3x + 5)(x - 3) = 5$$

151.
$$3x^2 - 7x + 1 = 0$$

152.
$$r^2 - 9 = 0$$

153.
$$(x-3)^2-25=0$$

154. The weight of a human fetus is described by the formula $W = 3t^2$, where W is the weight, in grams, and t is the time, in weeks, $0 \le t \le 39$. After how many weeks does the fetus weigh 1200 grams?

155. The alligator, an endangered species, is the subject of a protection program. The formula

$$P = -10x^2 + 475x + 3500$$

describes the alligator population, P, after x years of the protection program, where $0 \le x \le 12$. After how many years is the population up to 7250?

156. A building casts a shadow that is double the length of its height. If the distance from the end of the shadow to the top of the building is 300 meters, how high is the building? Round to the nearest meter.

P.9

In Exercises 157-159, graph the solutions of each inequality on a number line.

157.
$$x > 5$$

158.
$$x \le 1$$

159.
$$-3 \le x < 0$$



162.
$$(-1, \infty)$$

Solve each linear inequality in Exercises 163-168 and graph the solution set on a number line. Express each solution set in interval notation.

163.
$$-6x + 3 \le 15$$

164.
$$6x - 9 \ge -4x - 3$$

165.
$$\frac{x}{3} - \frac{3}{4} - 1 > \frac{x}{2}$$

166.
$$6x + 5 > -2(x - 3) - 25$$

167.
$$3(2x-1)-2(x-4) \ge 7+2(3+4x)$$

168.
$$7 < 2x + 3 \le 9$$

Solve each inequality in Exercises 169-171 by first rewriting each one as an equivalent inequality without absolute value bars. Graph the solution set on a number line. Express each șolution set in interval notation.

169.
$$|2x + 3| \le 15$$

170.
$$\left| \frac{2x+6}{3} \right| > 2$$

171.
$$|2x + 5| - 7 \ge -6$$

172. Approximately 90% of the population sleeps
$$h$$
 hours daily, where h is described by the inequality $|h - 6.5| \le 1$. Write a sentence describing the range for the number of hours that most people sleep. Do *not* use the phrase "absolute value" in your description.

173. The formula for converting Fahrenheit temperature,
$$F$$
, to Celsius temperature, C , is $C = \frac{5}{9}(F - 32)$. If Celsius temperature ranges from 10° to 25°, inclusive, what is the range for the Fahrenheit temperature?

Chapter P Test

1. List all the rational numbers in this set:

$$\{-7, -\frac{4}{5}, 0, 0.25, \sqrt{3}, \sqrt{4}, \frac{22}{7}, \pi\}.$$

In Exercises 2-3, state the name of the property illustrated.

$$2. \ 3(2+5) = 3(5+2)$$

3.
$$6(7+4) = 6 \cdot 7 + 6 \cdot 4$$

4. Express in scientific notation: 0.00076.

Simplify each expression in Exercises 5-11.

5.
$$9(10x - 2y) - 5(x - 4y + 3)$$

$$6. \ \frac{30x^3y^4}{6x^9y^{-4}}$$

7.
$$\sqrt{6r}\sqrt{3r}$$
 (Assume that $r \ge 0$.)

8.
$$4\sqrt{50} - 3\sqrt{18}$$

9.
$$\frac{3}{5+\sqrt{2}}$$

10.
$$\sqrt[3]{16x^4}$$

11.
$$\frac{x^2+2x-3}{x^2-3x+2}$$

12. Evaluate: 27^{-5/3}.

In Exercises 13-14, find each product.

13.
$$(2x-5)(x^2-4x+3)$$
 14. $(5x+3y)^2$

14.
$$(5x + 3y)^2$$

In Exercises 15-20, factor completely, or state that the polynomial is prime.

15.
$$x^2 - 9x + 18$$

16.
$$x^3 + 2x^2 + 3x + 6$$

17.
$$25x^2 - 9$$

18.
$$36x^2 - 84x + 49$$

19.
$$y^3 - 125$$

20.
$$x^2 + 10x + 25 - 9y^2$$

21. Factor and simplify:

$$x(x + 3)^{-3/5} + (x + 3)^{2/5}$$

In Exercises 22-26, perform the operations and simplify, if possible.

22.
$$\frac{2x+8}{x-3} \div \frac{x^2+5x+4}{x^2-9}$$

23.
$$\frac{x}{x+3} + \frac{5}{x-3}$$

$$24. \ \frac{2x+3}{x^2-7x+12} - \frac{2}{x-3}$$

25.
$$\frac{1-\frac{x}{x+2}}{1+\frac{1}{x+2}}$$

25.
$$\frac{1 - \frac{x}{x+2}}{1 + \frac{1}{x}}$$
26.
$$\frac{2x\sqrt{x^2 + 5} - \frac{2x^3}{\sqrt{x^2 + 5}}}{x^2 + 5}$$







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Find the solution set for each equation in Exercises 27-33.

27.
$$7(x-2) = 4(x+1) - 21$$

28.
$$\frac{2x-3}{4} = \frac{x-4}{2} - \frac{x+1}{4}$$

29.
$$\frac{2}{r-3} - \frac{4}{r+3} = \frac{8}{r^2-9}$$

$$30. \ 2x^2 - 3x - 2 = 0$$

31.
$$(3x - 1)^2 = 75$$

32.
$$x(x-2)=4$$

33.
$$\left| \frac{2}{3}x - 6 \right| = 2$$

Solve each inequality in Exercises 34-37. Express the answer in interval notation and graph the solution set on a number

34.
$$3(x+4) \ge 5x-12$$

35.
$$\frac{x}{6} + \frac{1}{8} \le \frac{x}{2} - \frac{3}{4}$$

35.
$$\frac{x}{6} + \frac{1}{8} \le \frac{x}{2} - \frac{3}{4}$$
 36. $-3 \le \frac{2x+5}{3} < 6$

37.
$$|3x + 2| \ge 3$$

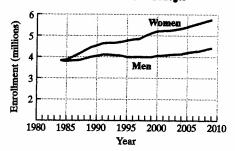
In Exercises 38-39, solve each formula for the specified variable.

38.
$$V = \frac{1}{3} lwh \text{ for } h$$

39.
$$y - y_1 = m(x - x_1)$$
 for x

40. The male minority? The graphs show enrollment in U.S. colleges, with projections from 2003 through 2009. The trend indicated by the graphs is among the hottest topics of debate among college-admission officers. Some private liberal arts colleges have quietly begun special efforts to recruit men-including admissions preferences for them.

Enrollment in U.S. Colleges



Source: U.S. Department of Education

The data for the men can be approximated by the formula

$$N = 0.01x + 3.9$$

where N represents enrollment, in millions, x years after 1984. According to the formula, when will the projected enrollment for men be 4.1 million? How well does the formula describe enrollment for that year shown by the line graph?

41. A vertical pole is to be supported by a wire that is 26 feet long and anchored 24 feet from the base of the pole. How far up the pole should the wire be attached?

In Exercises 35-37, find the domain of each logarithmic function.

35.
$$f(x) = \log_8(x+5)$$
 36. $f(x) = \log(3-x)$
37. $f(x) = \ln(x-1)^2$

In Exercises 38–40, use inverse properties of logarithms to simplify each expression.

38.
$$\ln e^{6x}$$
 39. $e^{\ln \sqrt{x}}$ **40.** $10^{\log 4x^2}$

- 41. On the Richter scale, the magnitude, R, of an earthquake of intensity I is given by $R = \log \frac{I}{I_0}$, where I_0 is the intensity of a barely felt zero-level earthquake. If the intensity of an earthquake is $1000I_0$, what is its magnitude on the Richter scale?
- 42. Students in a psychology class took a final examination. As part of an experiment to see how much of the course content they remembered over time, they took equivalent forms of the exam in monthly intervals thereafter. The average score, f(t), for the group after t months is modeled by the function $f(t) = 76 18 \log(t + 1)$, where $0 \le t \le 12$.
 - a. What was the average score when the exam was first given?
 - b. What was the average score after 2 months? 4 months? 6 months? 8 months? one year?
 - c. Use the results from parts (a) and (b) to graph f. Describe what the shape of the graph indicates in terms of the material retained by the students.

43. The formula

$$t = \frac{1}{c} \ln \left(\frac{A}{A - N} \right)$$

describes the time, t, in weeks, that it takes to achieve mastery of a portion of a task. In the formula, A represents maximum learning possible, N is the portion of the learning that is to be achieved, and c is a constant used to measure an individual's learning style. A 50-year-old man decides to start running as a way to maintain good health. He feels that the maximum rate he could ever hope to achieve is 12 miles per hour. How many weeks will it take before the man can run 5 miles per hour if c = 0.06 for this person?

3.3

In Exercises 44–47, use properties of logarithms to expand each logarithmic expression as much as possible. Where possible, evaluate logarithmic expressions without using a calculator.

44.
$$\log_6(36x^3)$$

45.
$$\log_4\left(\frac{\sqrt{x}}{64}\right)$$

$$46. \log_2\left(\frac{xy^2}{64}\right)$$

In Exercises 48–51, use properties of logarithms to condense each logarithmic expression. Write the expression as a single

logarithm whose coefficient is 1.
48.
$$\log_b 7 + \log_b 3$$

49.
$$\log 3 - 3 \log x$$

50.
$$3 \ln x + 4 \ln y$$

51.
$$\frac{1}{2} \ln x - \ln y$$

In Exercises 52-53, use common logarithms or natural logarithms and a calculator to evaluate to four decimal places.

3.4

Solve each exponential equation in Exercises 54-58. Express the answer in terms of natural logarithms. Then use a calculator to obtain a decimal approximation, correct to two decimal places, for the solution.

54.
$$8^x = 12,143$$

55.
$$9e^{5x} = 1269$$

56.
$$e^{12-5x} - 7 = 123$$

57.
$$5^{4x+2} = 37,500$$

$$58. e^{2x} - e^x - 6 = 0$$

Solve each logarithmic equation in Exercises 59-63.

59.
$$\log_4(3x-5)=3$$

60.
$$\log_2(x+3) + \log_2(x-3) = 4$$

61.
$$\log_3(x-1) - \log_3(x+2) = 2$$

62.
$$\ln x = -1$$

63.
$$3 + 4 \ln(2x) = 15$$

- **64.** The formula $A = 10.1e^{0.005t}$ models the population of Los Angeles, California, A, in millions, t years after 1992. If the growth rate continues into the future, when will the population reach 13 million?
- 65. The amount of carbon dioxide in the atmosphere, measured in parts per million, has been increasing as a result of the burning of oil and coal. The buildup of gases and particles traps heat and raises the planet's temperature, a phenomenon called the greenhouse effect. Carbon dioxide accounts for about half of the warming. The function $f(t) = 364(1.005)^t$ projects carbon dioxide concentration, f(t), in parts per million, t years after 2000. Using the projections given by the function, when will the carbon dioxide concentration be double the preindustrial level of 280 parts per million?
- 66. The formula $\overline{C}(x) = 15,557 + 5259 \ln x$ models the average cost of a new car, $\overline{C}(x)$, x years after 1989. When will the average cost of a new car reach \$30,000?
- 67. Use the formula for compound interest with n compoundings each year to solve this problem. How long, to the nearest tenth of a year, will it take \$12,500 to grow to \$20,000 at 6.5% annual interest compounded quarterly?

Use the formula for continuous compounding to solve Exercises 68–69.

- **68.** How long, to the nearest tenth of a year, will it take \$50,000 to triple in value at 7.5% annual interest compounded continuously?
- 69. What interest rate is required for an investment subject to continuous compounding to triple in 5 years?

