Math 3001 **Due Wed Nov 9** **Assignment #7**

1. Find a function to which the series converges poinwise. Does it converge uniformly on the whole domain?

   A)

$$\frac{x^2}{x^2+1}+\frac{x^2}{(x^2+1)(2x^2+1)}+\frac{x^2}{(2x^2+1)(3x^2+1)}+\cdots+\frac{x^2}{(nx^2+1)((n+1)x^2+1)}+\cdots.$$

   B)

$$\sum_{n=0}^{\infty}\frac{x^5}{(x^5+1)^n}.$$

2. Prove a Theorem that if a sequence of continuous functions converges uniformly on a set S to a function f, then f is continuous on S.

3. Using the Theorem about uniform convergence and continuity, determine whether or not the following sequence of functions converges uniformly on $x \in [0,1]$

   A)

$$f_n(x) = \frac{5}{3x^n+4}$$

   B)

$$f_n(x) = \frac{nx}{nx+4}$$

4. Find a sequence of functions (f_n) defined on $[0,1]$ such that each f_n is discontinuous at each point of $[0,1]$ and such that the sequence converges uniformly to a function f that is continuous on $[0,1]$.

5. Evaluate (with explainations)

$$\lim_{n\to\infty}\int_0^{\pi}\frac{n+\sin(nx)}{3n+\sin^2(nx)}\,dx.$$

6. Integrate the geometric series

$$\frac{1}{1-r} = 1+r+r^2+r^3+\cdots$$

   term by term from $-x$ to x, where $0 < x < 1$ to find the series for $\ln\frac{1+x}{1-x}$. Explain the reason why you can do this.

7. EXTRA POINTS

   If possible, find a sequence of functions (f_n) defined on $[0,1]$ such that each f_n is continuous at each point of $[0,1]$ and such that the sequence converges uniformly to a function f that is discontinuous at each point of on $[0,1]$.