1. Given the series $\sum_{n=1}^{\infty} a_{n}$ and $\sum_{n=1}^{\infty} b_{n}$, suppose that there exists a number $N$ such that $a_{n}=b_{n}$ for all $n>N$. Prove that $\sum_{n=1}^{\infty} a_{n}$ is convergent iff $\sum_{n=1}^{\infty} b_{n}$ is convergent.
Is it true that $\sum_{n=1}^{\infty} a_{n}=\sum_{n=1}^{\infty} b_{n}$ ?
2. Determine whether or not the series $\sum_{n=1}^{\infty}(\sqrt{n+1}+\sqrt{n})^{-1}$ is convergent? Justify your answer.
3. Let $\left\{x_{n}\right\}_{n=1}^{\infty}$ be a sequence of real numbers and let $y_{n}=x_{n}-x_{n+1}$ for all $n \geq 1$. Prove that the series $\sum_{n=1}^{\infty} y_{n}$ is convergent iff the sequence $\left\{x_{n}\right\}_{n=1}^{\infty}$ is convergent.
If the series $\sum_{n=1}^{\infty} y_{n}$ is convergent, what is the sum?
4. Find an example to show that the convergence of $\sum_{n=1}^{\infty} a_{n}$ and $\sum_{n=1}^{\infty} b_{n}$ not necessarily imply convergence of $\sum_{n=1}^{\infty} a_{n} b_{n}$.
5. Prove that if $\sum_{n=1}^{\infty}\left|a_{n}\right|$ converges and $\left\{b_{n}\right\}_{n=1}^{\infty}$ is a bounded sequence then $\sum_{n=1}^{\infty} a_{n} b_{n}$ converges.
6. a)Show by example that grouping of terms may change a divergent series to convergent.
b)Is (a) possible for a divergent series with all nonnegative terms?
c)Is it possible to change the sum of a convergent series by grouping of terms?
7. Show that the series

$$
1-\frac{1}{2}+\frac{1}{3}-\frac{1}{2^{2}}+\frac{1}{5}-\frac{1}{2^{3}}+\frac{1}{7}-\frac{1}{2^{4}}+\cdots
$$

is divergent. Why doesn't this contradict the Alternating Series Test?
8. Prove that if a series is conditionally converegnt, then the series of its negative terms is divergent.
9. Suppose that $\sum_{n=1}^{\infty} a_{n}$ is conditionally convergent series, and $s$ is a real number.
a)Explain why there exists a rearrangement of $\sum_{n=1}^{\infty} a_{n}$ that converges conditionally to $s$.
b)Is there a rearrangement of $\sum_{n=1}^{\infty} a_{n}$ that diverges?
10. EXTRA POINTS

Let $a_{n}>0$ for all $n \geq 1$. Let $b_{n}=\left(a_{1}+a_{2}+\cdots+a_{n}\right) / n$. Is the series $\sum_{n=1}^{\infty} b_{n}$ converegnt or diveregnt? Explain.

