Math 3001

- 1. Given the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$, suppose that there exists a number N such that $a_n = b_n$ for all n > N. Prove that $\sum_{n=1}^{\infty} a_n$ is convergent iff $\sum_{n=1}^{\infty} b_n$ is convergent. Is it true that $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} b_n$?
- 2. Determine whether or not the series $\sum_{n=1}^{\infty} (\sqrt{n+1} + \sqrt{n})^{-1}$ is convergent? Justify your answer.
- 3. Let {x_n}[∞]_{n=1} be a sequence of real numbers and let y_n = x_n x_{n+1} for all n ≥ 1. Prove that the series ∑[∞]_{n=1} y_n is convergent iff the sequence {x_n}[∞]_{n=1} is convergent. If the series ∑[∞]_{n=1} y_n is convergent, what is the sum?
- 4. Find an example to show that the convergence of $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ not necessarily imply convergence of $\sum_{n=1}^{\infty} a_n b_n$.
- 5. Prove that if $\sum_{n=1}^{\infty} |a_n|$ converges and $\{b_n\}_{n=1}^{\infty}$ is a bounded sequence then $\sum_{n=1}^{\infty} a_n b_n$ converges.
- 6. a)Show by example that grouping of terms may change a divergent series to convergent.b)Is (a) possible for a divergent series with all nonnegative terms?c)Is it possible to change the sum of a convergent series by grouping of terms?
- 7. Show that the series

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{2^2} + \frac{1}{5} - \frac{1}{2^3} + \frac{1}{7} - \frac{1}{2^4} + \cdots$$

is divergent. Why doesn't this contradict the Alternating Series Test?

- 8. Prove that if a series is conditionally convergent, then the series of its negative terms is divergent.
- 9. Suppose that ∑_{n=1}[∞] a_n is conditionally convergent series, and s is a real number.
 a)Explain why there exists a *rearrangement* of ∑_{n=1}[∞] a_n that converges conditionally to s.
 b)Is there a rearrangement of ∑_{n=1}[∞] a_n that diverges?

10. EXTRA POINTS

Let $a_n > 0$ for all $n \ge 1$. Let $b_n = (a_1 + a_2 + \dots + a_n)/n$. Is the series $\sum_{n=1}^{\infty} b_n$ convergent or divergent? Explain.