Math 3001 **Due Fri Sept 16** **Assignment #1**

1. Find the limit as $n \to \infty$ of the sequence given by its general term, or explain why the limit does not exist.
   (a) $\dfrac{n^3 + n^2 + 1}{1 - n^2 - n^3}$
   (b) $\left(\dfrac{2+n}{n}\right)^{3n}$
   (c) $\cos(\pi n)$
   (d) $\left(\dfrac{n}{3+n}\right)^{n}$
   (e) $3n \sin\left(\dfrac{2}{n}\right)$

2. Prove that each of the following sequences (given reccursively) is convergent, and find the limit.
   (a) $a_{n+1} = (6 + a_n)^{1/2}$, $a_1 = \sqrt{6}$
   (b) $a_{n+1} = \dfrac{a_n}{2} + \dfrac{x}{a_n}$, where $x > 0$ and $a_1 > 0$ are arbitrary real numbers.

3. Show that if $\lim_{n\to\infty} a_{2n} = \lim_{n\to\infty} a_{2n+1} = L$ then the sequence $(a_n)_{n=1}^{\infty}$ is convergent to L.

   Is it always true that if two given subsequences of a sequence have the same finite limit then the sequence converges?

4. A sequence is said to be contractive if there is such a number $1 > k > 0$ that

$$|a_{n+2} - a_{n+1}| < k|a_{n+1} - a_n|.$$

   Prove that every contractive sequence is a Cauchy sequence.

5. Show that $\limsup\,(a_n + b_n) \leq \limsup a_n + \limsup b_n$ for any bounded sequences.

6. True or False?
   (a) a bounded sequence is convergent if and only if it is monotone
   (b) every sequence has a convergent subsequece
   (c) every cluster point is a limit point
   (d) every limit point is a cluster point
   (e) every bounded and divergent sequence has at least two cluster points
   (f) limit superior is always greater that limit inferior
   (g) every contractive sequence of real numbers converges

7. **Extra Points Problem**

   Give an example of a sequence which has a countably infinite collection of cluster points.