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Geometry as a Key Component in the Education of a Skillful Designer

Margo Kondratieva, Memorial University, Canada

Abstract: Geometry has often been used, and its advance as a science stimulated, by architectural and artistic design. In this paper I discuss examples of enriching geometrical activities that could play a valuable role in the development of flexibility and precision of visual thinking, which is essential for education of a skillful designer.

Keywords: Mathematical Education, Visual Thinking, Spacial Imagination, Modelling and Graphic Representation

Introduction

RENÉ DESCARTES PROCLAIMED that there is no object that is easier for our mind to handle than a geometrical figure. However, this is not universally true for everyone. Visual thinking and imagination are very important abilities required in many professions such as engineering, architecture, and design. But what exactly constitutes visual thinking? How does one form visual images which allow him/her to manipulate them mentally with a high degree of precision? What educational and life experiences support such development?

In order to approach these questions we first note that one has to distinguish the *visual appearance* of an artifact or object from its *visual mental image*. The former only conveys external characteristics and is connected with our visual perception of the form, while the latter also takes into account the internal logical structure of the object. For example, saying that "architectural design focuses on visual thinking, that it uses visual thinking in order to transform and compile spatial solution" (Barelkowski, 2010, p.127) one conveys a significantly different message depending on whether one talks about visual thinking in terms of visual appearances or in terms of visual mental images. Indeed, a more common narrow understanding of visual thinking in terms of visual appearances motivated Barelkowski "to argue with the predominate perception of architectural design mainly as a result of visual thinking" (ibid). He warns against such interpretation of visual thinking that largely ignores (a) its verbal and reasoning mechanisms and (b) the fact that both visual and verbal models are indispensible and practically inseparable. The view of visual thinking as a non-verbal way to process information (Brawne, 2003) frequently generates the impression that visual thinking lacks explicit reasoning and logic. This view may be contrasted with Arnheim's position that interpreting visual information is a complex intellectual act requiring abstraction, reasoning, and awareness of structure, and that drawing itself "is a form of reasoning, in which perceiving and thinking are indivisibly intertwined" (Arnheim, 1969).

The observation discussed above shades the light on the controversy related to the Cartesian statement. For Descartes and other mathematically inclined minds, a geometrical figure is

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not simply a picture; it has a structure -- an underlying logic and hidden geometrical properties. The image itself is just the tip of the iceberg which may hold a large amount of additional information available only to an experienced viewer. In this sense, "a *geometrical figure* may be described as having *intrinsically* conceptual properties. Nevertheless, a geometrical figure is not a mere concept. It is an image, a visual image. …it includes the mental representation of space property" (Fischbein 1993, p.141). The possibility to grasp the message encoded by few curves drawn on a piece of paper comes from previous practice with drawing and reflecting on their geometrical properties, and thus deeper understanding and internalization of these properties.

In this respect, it seems unfortunate that many students of the arts often lack basic geometrical knowledge and experiences such as construction with a compass and straightedge (Hann & Thomas, 2008). In the appendix of their paper, Hann & Thomas propose several exercises that help to fulfill this gap for future designers. My position is that it may be too late to present students with many of these exercises (e.g. construction of a regular pentagon and hexagon) at the senior undergraduate university level. Many of these activities naturally belong to the earlier school experience and due to their fundamental character and developmental importance for children they should be presented to all learners throughout their primary and secondary school education in geometry.

This paper has the following structure. In the first section it discusses in more detail the process of formation of visual mental images. The second section explains the notion of an interconnecting problem (Kondratieva, 2011a) in the context of a students' continuous development through their primary, secondary, and tertiary levels of education. The third section gives some examples of activities consistent with the concept of "interconnecting problem" and its application to the education of designers.

Formation of Visual Thinking in Terms of Visual Mental Images

Many types of design presuppose visual thinking. In this respect, the process of forming visual mental images is of importance. Indeed, how are visual mental images formed? Following the classical ideas of Jean Piaget, Jerome Bruner, and Dina and Pierre van Hiele, we summarize the most important steps of visual-spatial development as follows.

At first, a child uses *signifiers* as indicators of (signified) objects, indistinguishable from them. Then s/he acquires realization of existence of images (signifiers) independently from objects, and develops the ability to recognize objects by their images. Next, the child forms rough visual images which are mostly static and reproductive. Their focus is on external attributes. At this point, the ability to recall or evoke an image, and to produce naïve drawings, is formed -- manifesting a half-way progress between imitation and representation in physical action to representations in thoughts. When a child notices a structure, and is able to refocus on the internal properties of the image, this enhances the child's ability to reconstruct a physical picture from the parts and to move an image in thoughts. Next, the formation of the ability to act on a picture (e.g. to recognize individual components, and transform, or draw additional elements) can be observed. At this time, the child starts making conjectures, forms *anticipatory* images and becomes capable of informal deductive reasoning based on image's properties. At this stage, drawing geometrical and schematic pictures can be viewed as an attempt to organize information. Finally, a child can form visual mental images equipped with a network of equivalent properties and reveal a conceptual grasp of objects and events.

This provides the foundation for abstraction from images as symbols of physical objects to signs and the ability to (easily) switch between visual and verbal-logical modes.

As a child grows s/he learns to deal with and distinguish between *physical objects*, *graphical pictures of physical objects*, *visual mental images*, *or figural concepts*. Some of these notions have a blending character and resemble cognitive shifts in the process of conceptual growth. For instance, a *graphical picture* is itself a physical object because one can interact with it in a physical way, but it also is an icon which represents another physical object in a schematic way and in this sense is just an image of it. Similarly, *figural concepts*, also known as *geometrical figures* are "mental entities which possess simultaneously conceptual and figural characteristics" (Fischbein, 1993). *Visual mental images* emerge from an attempt to grasp the sense of the reality and are always a simplification of it. They can be formed through a child's experience with physical objects, as well as with graphical pictures and geometrical figures. Thus, visual mental images may contain both empirical and theoretical information derived through experimentation with geometrical figures.

Researchers have hypothesized about why visual mental images are formed. According to their observations, a very young child perceives objects entirely through his actions on them. At first, a child may even reject the very existence of objects which are out if his reach or view. Terms *sensorimotor* or *enactive* are used in order to characterize a concrete, hand-on phase of a learner's representation of the world. This form of representation is found in young children, but also used by adults when the task calls for it.

A pictorial or *iconic* representation of an object is formed later, when a series of actions are organized in a summative image, which becomes abstract and free of concrete actions. And even then the child still depends on specific representations of the world through his actions. A similar phenomenon of *iconization of concrete actions* was observed in adult subjects, who constructed a mental plan of a labyrinth in which they repeatedly were searching for a way out (Mandler, 1962). The iconic stage assumes the passage from naïve and concrete drawings towards mental imagination, which provides the necessary ground for the next stage, when the learner operates on a formal and abstract level. "The adult, as does the child, needs a system of signifiers dealing not with concepts but with objects as such and with the whole past perceptual experience of the subject. This role has been assigned to the image" (Piaget & Inhelder, 1969, p.70). Thus, images (icons) are not exact copies of perceived objects, but rather meaningful re-creations of perceptions. "Formation of mental images cannot precede understanding" (ibid, p.72). Different types of understanding apparently correspond to different degrees of insight of the images. Instrumental or algorithmic understanding helps a learner to formally redraw an image, while conceptual understanding underlines various geometrical properties of the image and connections between them.

The ability to preserve and reconstruct visual mental images depends on the ability to focus on essential and invariant properties of the physical objects they represent. While saying that "all perceiving is also thinking, all reasoning is also intuition, all observation is also invention", Arnheim (1969) suggested that visual information has its own structure and cannot be transformed simply in a form of picture unless this structure is also conveyed to the learner and the meaning of the drawing is negotiated. In order to perceive and mentally manipulate this intrinsic geometrical structure of an image the learner requires a gradual development of corresponding *schema*. Kant describes schemata as rules that allow the imagination to mentally construct or trace a general geometrical form that gives the pure sensible concept meaning. "No image of a triangle would ever be adequate to the concept of it. For it would not attain the generality of the concept, which makes this valid for all triangles…" But images must be connected with the concept "always only by means of the schema that they designate" (Kant, 1998/1781, p.273, A141-142). Research in embodied cognition hypothesizes that since they are formed from our bodily interactions and linguistic experience, image schemata participate in cognitive processes such as the establishment of patterns of understanding and reasoning. Johnson (1987) and Lakoff (1987) suggest that schemata are particularly important for visual-geometric reasoning. A mismatch between image schemata and formal definitions (verbal descriptions) of geometrical concepts has been identified as a source of difficulty in the study of mathematics and related subjects. Once formed, such a mismatch can be persistent and difficult to correct. Therefore, it is important to ensure that students receive early geometrical experiences with proper coordination of visual and verbal components. Such experiences are particularly significant in the education of future designers whose many practices involve, and rely upon, elements of geometrical vision.

Interconnecting Problems: The Complex Needs of Designer and Pure Geometrical Tasks

In geometrical instruction, one way to reflect this long-term continuous development of visual thinking is to consider the notion of an *interconnecting problem*. An interconnecting problem is characterized by the following properties (Kondratieva 2011a): (1) It allows simple formulation; (2) It allows various solutions at both elementary and advanced levels; (3) It may be solved by various mathematical tools from distinct mathematical branches, which leads to finding multiple solutions, (4) It is used in different grades and courses and can be discussed in various contexts.

It is proposed (Kondratieva 2011a) that a long-term study of a progression of mathematical ideas and images revolved around one interconnecting problem is useful for developing a comprehensive perception of corresponding geometrical entities. Due to the wide range of difficulty levels of its solutions, the same interconnecting problem may appear at the elementary school level, and then in progressive grades at the secondary or even tertiary level. The students, familiar with the problem from their prior hands-on experience, will use their intuition to support more elaborate techniques and theoretical approaches presented in the upper grades. In this way, an interconnecting problem may support development of visual mental images of the objects involved in the problem as well as corresponding schemata which would allow manipulating with them mentally.

An interconnecting problem may serve the needs of certain professions such as a designer. A problem, taken from a particular context, for instance architectural or textile design, shall be analyzed from the point of view of more elementary subtasks that can be learned and performed by the student at the primary or secondary level in order to prepare them for more professional and focused education at the tertiary level. Adopted for the more elementary levels, the problem shall be freed from particular technical details and at the same time must preserve its original description and contextual position in order to develop a students' interest and show them possible applications of mathematics in real life situations. An adequate past experience and knowledge of geometrical properties will ensure proper manipulation of these visual images in designing new artistic forms.

The creation of an interconnecting problem undergoes the following stages: (A) choosing an initial question; (B) tailoring questions to elementary approaches; (C) upgrading to more advanced techniques; and (D) finding contexts suitable for the identified approaches and techniques within the overall curriculum (Kondratieva 2011b). When an interconnecting problem is created with a particular aim of further education in design, the initial choice is dictated by both the importance of the problem in design education and the value of the identified subtasks for geometrical education. The next section presents several sample problems suitable for primary or secondary level mathematics courses and valuable for education of future designers in particular.

Sample Problems in Geometry for Future Designers

The Golden Ratio

The following assignment is motivated by Exercise 6 taken from Hann & Thomas (2008): Conduct a geometrical analysis of an image (e.g. face, building) with the objective of establishing symmetries and the presence of proportions which conform to the golden ratio. Hann & Thomas (2008) propose this exercise within the framework of a university level course "Design Theory - Structure and Form" and argue that "gaining a basic understanding of geometrical concepts … students shall be able to conduct structural analyses of … images, paintings, sculpture, patterns, tilings and other forms of two- and three- dimensional designs" (p. 16). In this section, I will illustrate that at the more elementary level, one can gain many of these geometrical concepts. Considering the above problem as an interconnecting one, we can see how its various subtasks may be approached at the secondary school level with a focus on geometry.

Golden section (ratio) is a number with a curious algebraic property: its reciprocal differs by 1 from the number itself. Traditionally, the golden section is denoted by Φ and its reciprocal by $\phi = \frac{1}{\Phi}$, so we have $\Phi - \varphi = 1$. Solving this relation we find that $\Phi = \frac{1+\sqrt{5}}{2} \approx 1.618..$, $\phi = \frac{\sqrt{5}-1}{2} \approx 0.618..$ It follows that the following relation $\varphi^n = \varphi^{n+1} + \phi^{n+2}$ holds for all n ≥ 0. In particular case (n = 0) we have $\varphi + \varphi^2 = 1$, which is in the agreement with $\varphi \cdot \Phi = 1$.

The golden rectangle is also a mathematically well-defined object. By definition, it has dimensions *a* and b (b is greater than a) such that if one cuts from it a square of size *a.* × *a*, the remaining rectangle becomes similar to the original one. Figure 1 shows two golden rectangles ABGH and FEGH, while ABEF is a square. This property allows setting up and finding the proportion *b* : *a*, which appears to be the golden section Φ. The fact that the second rectangle is similar to the original one ensures that the process of cutting squares and getting golden rectangles of smaller sizes is virtually infinite.

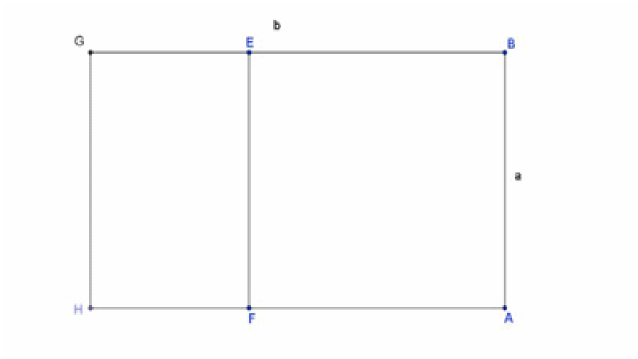

Figure 1: Golden Rectangle and Similarity

The problem of setting up the equation for golden section and solving it in radicals naturally belongs to the secondary school algebra. Its solution may be presented in the following simple way. By definition of golden rectangle we have $\frac{b}{a}=\frac{a}{b-a}$. Denote $x=\frac{b}{a}$, to rewrite $x=\frac{1}{x-1}$. Thus x satisfies the equation $x^2 - x - 1 = 0$. Solving this quadratic equation we obtain two roots. The positive root is of interest, which gives the exact value $x=\Phi=\frac{1+\sqrt{5}}{2}$.

A geometrical construction task follows immediately from this irrational expression. *How to construct a golden rectangle precisely?* The answer comes from the consideration that $\sqrt{5}$ is the diagonal of a rectangle with sides 2 and 1. Thus dividing a square ABEF into two rectangles where their height is twice as their width, and using compass and an unmarked straightedge one can perform the construction as shown in Figure 2. This again can be done within the secondary school curriculum in plain geometry.

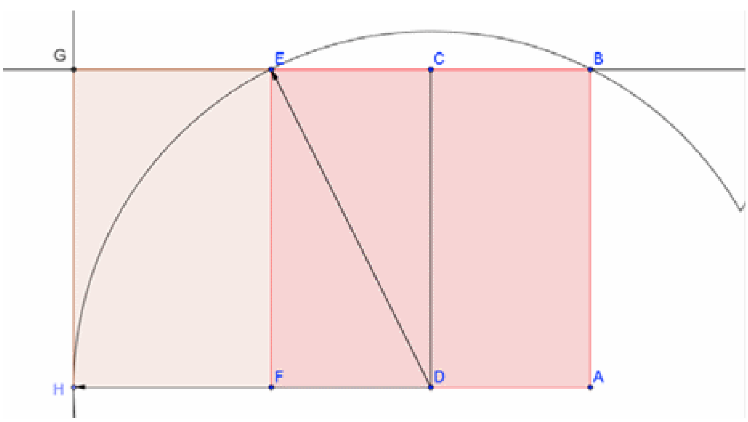

Figure 2: Construction of a Golden Rectangle with a Compass and Straightedge

At the same time, the secondary school curriculum can be enriched by familiarizing students with examples of golden proportions in modern art and architecture as well as asking them to investigate various arguments of proponents and opponents of the “golden section approach" to ancient art analysis. Letting the students themselves decide to what extend they find convincing the idea that Egyptian, Greek or Italian artists employed golden proportion, mathematics teachers at the same time can discuss important mathematical topics such as measurement and approximations.

The idea of approximation of the golden rectangle can also be illustrated by the Fibonacci rectangles, that is, rectangles with dimensions equal to a pair of consequent Fibonacci numbers, for example $a = 5$, $b = 8$ or $a = 8$, $b = 13$. Figure 3 shows few Fibonacci rectangles that are composed from squares of side 1, 2, 3, 5, 8.

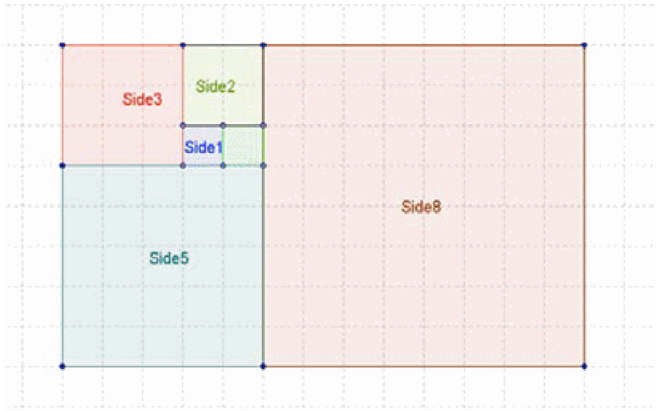

Figure 3: Fibonacci Rectangles as an Approximation to the Golden Rectangle

This representation clarifies the link between golden rectangle, the famous Fibonacci *sequence* and the *limit* closely related to it. The topic can be discussed in a pre-calculus course in senior secondary level. Recall that the Fibonacci sequence starts with 1, 1 and is given by the rule that each next term is the sum of two previous, that is 1, 1, 2, 3, 5, 8,… The key property of our interest is that taking the ratios of two consecutive terms one approximates the golden section and the precision of the approximation increases as the terms grow. In mathematical symbols we write *limit* $F_{n+1}/F_n = \Phi$, when $n \to \infty$.

Secondary school geometry may incorporate more elaborate construction than a golden rectangle. Two examples are given in Figures 4 and 5. In order to draw the octagram presented in Figure 4, one starts with a square ABCD and places points E and F on side AB such that to obey the golden section proposition, AB:AF=BA:BE= Φ. Points G, H, I, J, K, L are placed on the remaining three sides of the square in a similar manner. Then one connects the vertices of the square with new points on its sides by segments, as shown in Figure 4. It can be proved that four points A_1, B_1, C_1, D_1 resulting from the intersection of these segments are vertices of a square. The same procedure can be applied to this new square and thus producing an infinite sequence of similar octagrams. This construction is found among ancient Egyptian drawings and is believed to be used as an etalon to measure body proportion in Egyptian arts.

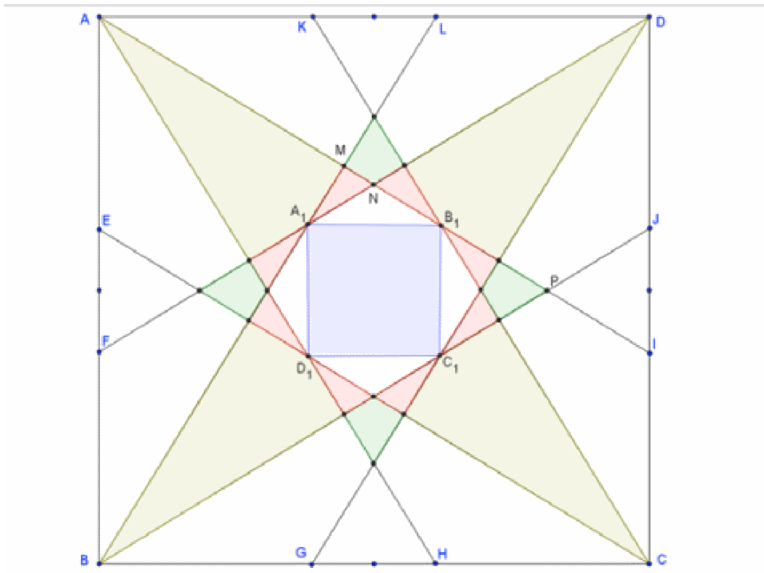

Figure 4: Egyptian Octagram

The concepts of similarity and golden section are central in the following set of exercises in Euclidean geometry involving the Egyptian octagram (see e.g. Voloshinov, 2000).

1. Redraw the Egyptian octagram starting from a square ABCD.
2. Prove that A_1,B_1,C_1,D_1 is a square. Draw a similar figure inside this square. Observe that the process is infinite.
3. Find triangles similar to ABL.
4. Show that BMP is a right triangle and find triangles similar to it.
5. Find segments whose length is expressed in terms of powers of golden section Φ^n, where n is an integer number.

Pythagorean pentagrams and pentagons present another opportunity to reveal the appearance of golden section and similarities in plane geometry (see Figure 5).

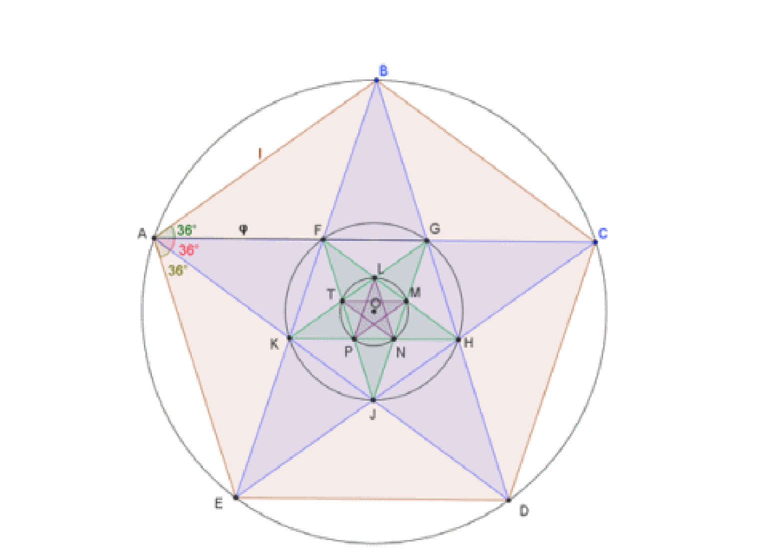

Figure 5: Pythagorean Pentagram

1. Show that diagonals of a regular pentagon are divided by the point of intersection in golden ratio, e.g. AG: GC = Φ.
2. Show that the sides of regular pentagon and corresponding pentagram are in golden proportion AB:AF = Φ.
3. Show that vertices FGHJK of a pentagram form a regular pentagon such that its side is Φ^2 times smaller that the side of original pentagon: AB: FG= Φ^2.
4. Observe that side lengths of the infinite sequence of pentagons (or pentagrams) form a geometric sequence $1, q, q^2, q^3, \ldots$ with geometric ratio $q = 1/\Phi^2$.
5. Show that segments of a pentagram are connected by various mean values, for example $AG = \frac{AC+FG}{2}$ arithmetic mean; $AG = \sqrt{AC \cdot AF}$ $AF = \sqrt{AG \cdot FG}$ geometric mean; harmonic mean $AF = \frac{2}{1/AC + 1/FG}$.
6. Show that if AB=1 then AF=GC= ϕ and AC =Φ.
7. Show that in a isosceles triangle XYZ, in which base angles $\hat{Y} = \hat{Z}$ are twice bigger than angle $\hat{X}$, the angular bisector YV divides the opposite side in the golden ratio: XV: VZ= Φ (An example of such triangle may be found in a pentagram, for instance ACD).

The richness of mathematical facts related to a pentagram was known from ancient times. That is why Pythagoreans had chosen this figure to be a symbol of life, beauty and health. Medieval scientist Luca Pacioli described many of pentagram's properties with admiration in his book about proportions "De divina proportione" written in 1497. Obviously, the geometry of pentagram presents interest for mathematicians. Thus is it tempting for historians of architecture and art try to find proportions based on the golden section or similar principles in the work of famous artists and architects. What are the secrets of proportions of magnificent Parthenon or mysterious Mona Lisa's portrait? [1]

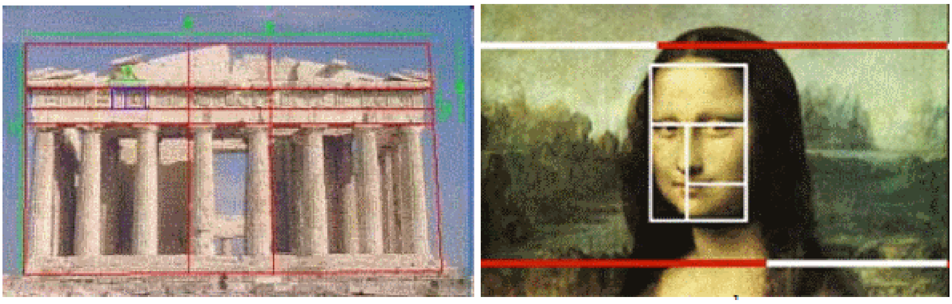

Figure 6: The Search for the Golden Rectangle in Art and Architecture

The search for aesthetic theories of mathematical bases of artistic perfection has a long history. For instance, the book "Kanon", written by Polykleitos, did not survive but some principles can be revealed by examining survived copies of his famous statue Doryphoros (450-440 B.C.). The ideas of Iktin, the creator of the Parthenon, can be reached only implicitly through the Ten Books of Vitruvius, the handbook of architecture of his time. Handwritten documents with drawings by Leonardo Da Vinci also witness the many attempts of this ingenious scientist and artist to systematize and connect his measurements of human body and a sense of human beauty. Four books about proportions by Albrecht Durer explicitly describe his method of drawing based on exact measurements, sense of proportions and similarity, his search for a universal method of drawing a human body, and the possible geometrical transformations of his drawings. Books and artifacts like those listed above transcend the boundaries between science and art and may serve as a rich source of geometrical problems inherited from the history of the progress of humanity. Obviously, there are several contemporary recourses aimed at summarizing past experiences in view of the modern development that can be adopted in today's classroom (see e.g. Elam, 2001 and Levy, 2005).

The golden section surprisingly appears in other design-related topics. For instance, the periodic tiling or tessellation of a plane using one or several regular polygons is of interest for textile designers. In the periodic case, patterns are comprised of motifs that repeat on regular basis across the plane. For instance, one can build a tiling of a plane without gaps and overlaps from either regular triangles, or squares or hexagons. This type of tiling has a translational symmetry. Aperiodic or Penrose type tilings which do not have a translational symmetry can be produced from a kite and dart that are shown on Figure 7. Note that the kite has angles 72, 72, 72 and 144 degrees, while the dart has angles 36, 36, 72, and 216 degrees, which ensures that the sides of each figure are in golden ratio. These two basic modules produce, besides periodic tilings, several aperiodic tilings.

[1] These and other related images can be found at the following links (accessed April 20, 2011)
http://www.math.vt.edu/people/gao/math/godensection/
http://coditootsiesleefant.blogspot.com/2009/11/math-is-used-in-art-all-time.html

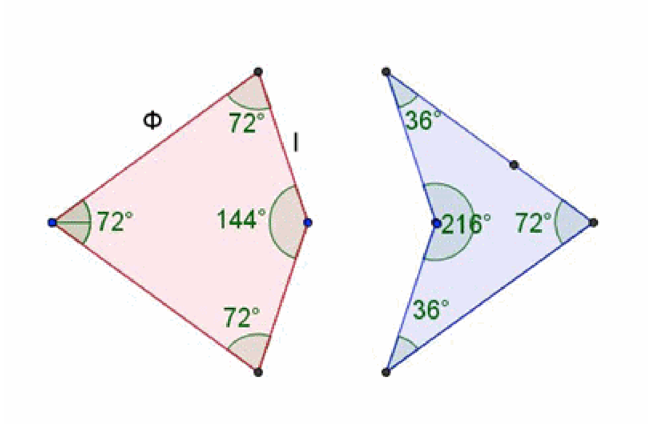

Figure 7: Penrose's Kite and Dart – Two Basic Modules to Produce an Aperiodic Tiling

Two More Ideas for Interconnecting Geometrical-design Problems

Tracery in Gothic architecture is another point of connection between geometric experiences and the training of art/design students. Artmann (1991) proclaims that tracery is "the most mathematical kind of art known to me". Indeed, just re-drawing a Rose window, one of which is shown in Figure 8 (left), students meet many geometrical questions such as "how to divide a circle in N equivalent parts?" or "how to ensure that certain lines and circular arcs are either tangent to each other or continuously extending each other?". Such exercises help students to understand what is possible in geometrical drawing. The creation of new forms requires precise calculations as well as a sense of harmony. A task appropriate for a course in elementary geometry which introduces students in this topic is as follows: **"**Let points A, B, C, and D be placed on a circumference or radius R such that the arcs AB, BC, CD, and DA are of the same length. Four semi-circular arcs with the same radius R and centers at A,B,C, and D respectively are drawn such that their end point meet at points K,L,M, and N as shown on Figure 8 (right). Find the area, shaded on the figure, that is bounded by the circumference and a curve formed by the four arcs. Your answer should be expressed in terms of radius R."

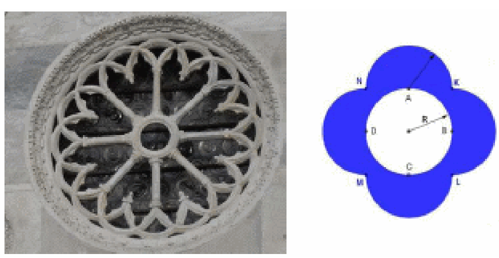

Figure 8: Rose Window (Left) and a Task from Elementary Geometry (Right)

One can see that the area is equal to the areas of square with side 2R and the circle of radius R. Thus the answer is $(4 + \pi)R^{2}$. More advanced problems are considered in (Artmann, 1991) with examples of Gothic window drawings. A “Handbook of Ornaments” written by Franz Sales Meyer in 1898 also contains many potential ideas for both a teacher of geometry and a mentor of future designers.

The final example which I would like to mention in this paper is regular Platonic and semi-regular Archimedean solids. It was noted that 3D geometry presents more difficulties for students and that students receive insufficient practice in their geometry courses so that their intuition about 3D space remains largely undeveloped (see e.g. Tanguay and Grenier, 2010). At the same time, problems involving polyhedra are of interest for designers. For instance, Hann & Thomas (2008) refer to the problem of finding arrangements similar to the 2D-tessalation mentioned above, but now considered in 3D space and involving regular or semi-regular solids placed without gaps or overlaps. The appearance of solids and their meaning in ancient and medieval architecture is of interest from both the historical and cultural points of view. For instance, examples of various appearances of *cuboctahedron* in 13th century art in the Turkey region allowed Hisarligil & Hisarligil to regard geometry in architecture of Seljuk Turks as the “metaphoric language of a poetry manifesting cosmic design” (Hisarligil & Hisarligil, 2009, p.123). Current investigation in architecture and its history may fuel secondary school mathematics curriculum and supply many interesting problems lying on the edge between 3D geometry and design, making the study of 3D geometry more interesting and helping the students receive adequate geometrical experiences at earlier stages of their education.

Conclusion

As a science, Geometry emerged from various human practices in response to their needs in construction, navigation and even beautification. “To produce decorations for their weaving, pottery, and other objects, early artists experimented with symmetries and repeating patterns” (Henderson & Taimina p. 1). This led to the study of tiling, symmetries, group theory. They “also explored various methods of representing physical objects and living things”, which led to the study of perspective and projective geometry. “As humans built shelters, altars, bridges … they discovered ways to make circles of various radii, and polygonal/polyhedron structures. In the process they devised systems of measurement and tools for measuring” (ibid, p. 4). Thus, the development of geometry especially at its earlier stages has many points of connection with the development of design as a part of human activity. Design called for practical answers from geometry, while geometrical constructions in their turn stimulated new ideas of design. This fact should not be ignored, and instead it should be used in order to inspire pedagogical practices in both areas.

In this paper I have presented several examples aiming to show that the secondary school curriculum may offer invaluable experiences for future designers. At the same time some problems adopted from the professional designers’ education are capable of enriching secondary school mathematics, bringing more real-life, history and art-related projects and thus creating interest in the students for learning mathematics with applications in these areas. Thus, it seems to be important for curriculum developers and teachers at different levels of education to collaborate and exchange their professional knowledge in order to single out appropriate and interesting tasks for secondary school students. Within this approach, a student's development is supported in a more continuous and logically-structured way by offering these tasks at earlier stages of student's education.

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About the Author

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Margo Kondratieva received a PhD degree in mathematics in 1994. Margo Kondratieva has about 15 years of teaching experience in undergraduate mathematics. Her current research interests are in the area of mathematics education, formation of mathematical thinking and learning, and design of educational experiences enriching students understanding of the subject. Margo Kondratieva has about 20 publications in peer reviewed journals in the area of mathematics and education. Margo Kondratieva has attended about 50 international conferences, including International Congress of Mathematicians in 1994 and in 2002, European Congress of Mathematics in 1996 and in 2004, as well as several studies conducted by the International Committee for Mathematical Instructions.

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