



Geometrical sophisms and mathematical proofs



ICMI Study 19



Margo Kondratieva

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Truth and truth-likeness



...as perceived from



substantive **mathematical meaning**

or



adequacy to **physical model**.



Proving and trusting



★ For some students proving is a formal ritual;



★ For many students trusting or believing [a statement] comes with examples, figures, analogies etc.





Deductive thinking explained via:



★ Factual knowledge (recall by analogy)
...but how we derive knew knowledge?



★ Formal rules (logic)
... but does it reflect the process?



★ Mental Models



Visual Mental models

- ★ Are essential for deductive reasoning;
 - ★ Each model represents a *possibility*;
 - ★ The structure of the model captures the common features of different possible representations of the model;
- (Johnson-Laird, Byrne, Polk, Newell)*





Theory of Visual Mental Models

- ★ mental models explicitly represent only truth (memory is saved by not showing false statements);
- ★ explains deductions about *what is probable* (probability is defined by the number of mental models which are in agreement with the conclusions).





Deduction is a justified conclusion.



- ★ If I am hungry then I have a snack;
- ★ If I have a snack then I have a light dinner



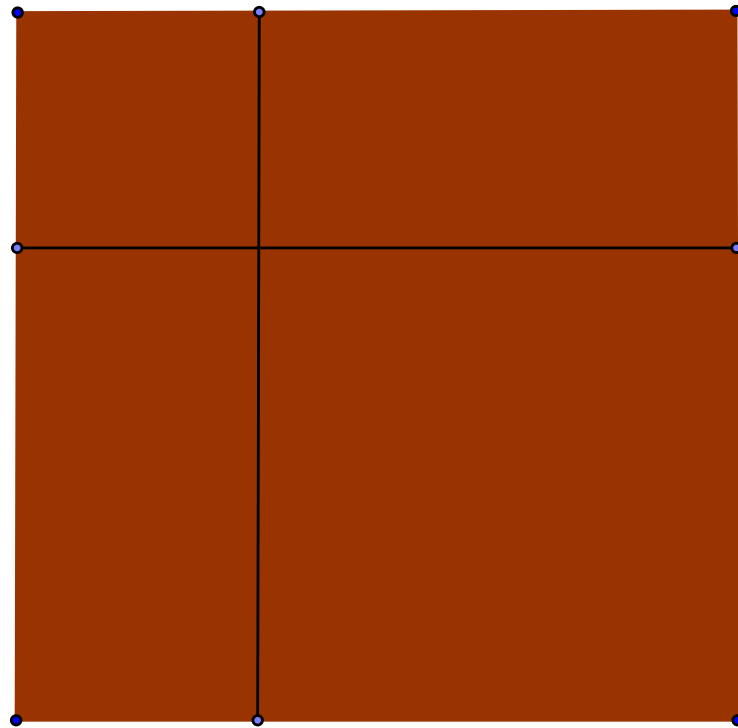
- ★ **If I am hungry then I have a light dinner!**





$$(a+b)(a+b)=aa+2ab+bb$$

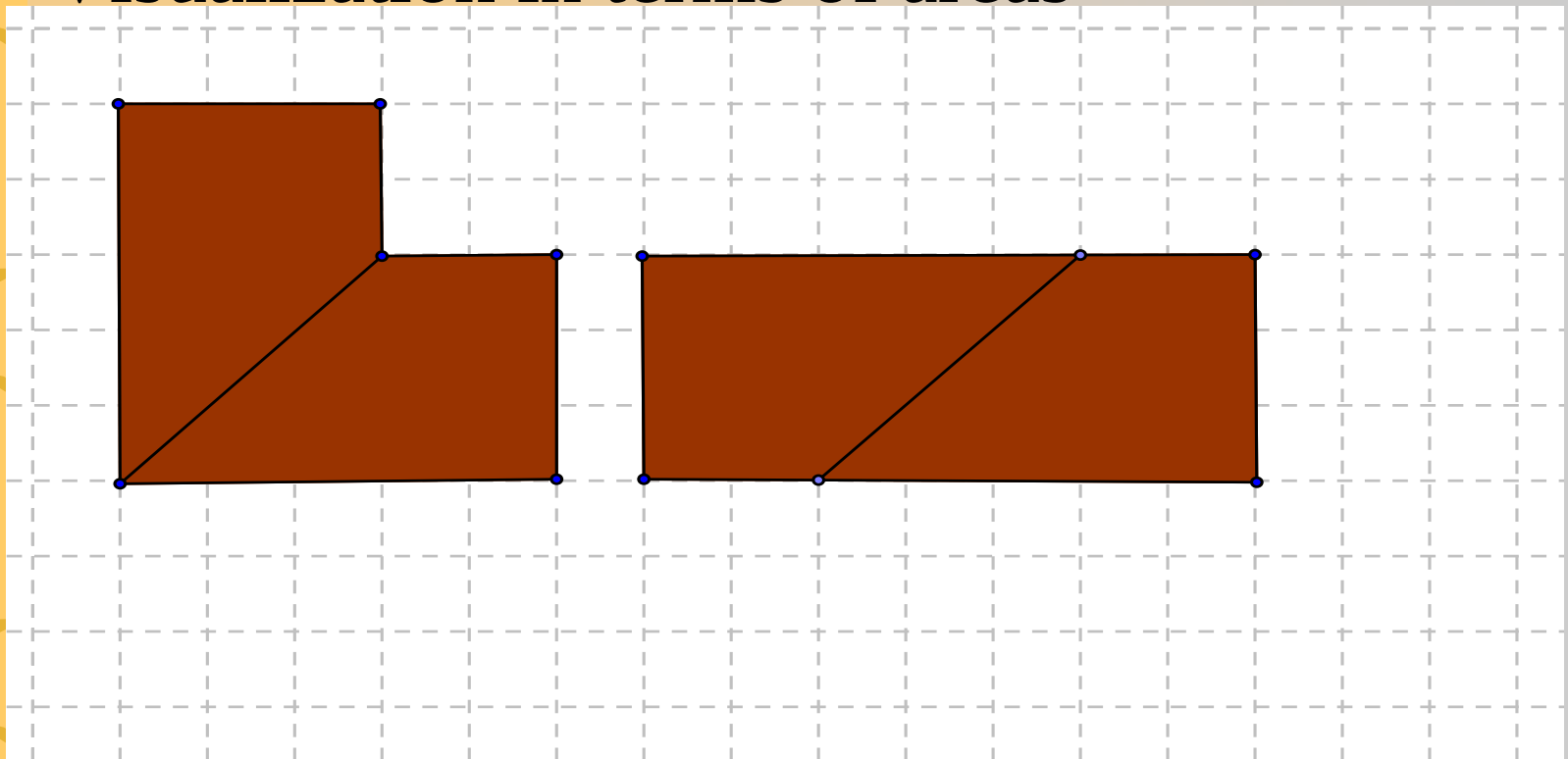
Visualization: area of the square





$$aa-bb=(a+b)(a-b)$$

Visualization in terms of areas





Pictures are good but...



- ★ May not cover all cases (e.g. negative a,b)
- ★ They are not *precise* (in Platonic sense).



- ★ Simply presenting picture will not transfer an idea to the learner. One needs to explain the essential features and structure of the image. (*Arnheim, Visual thinking. 1969*)





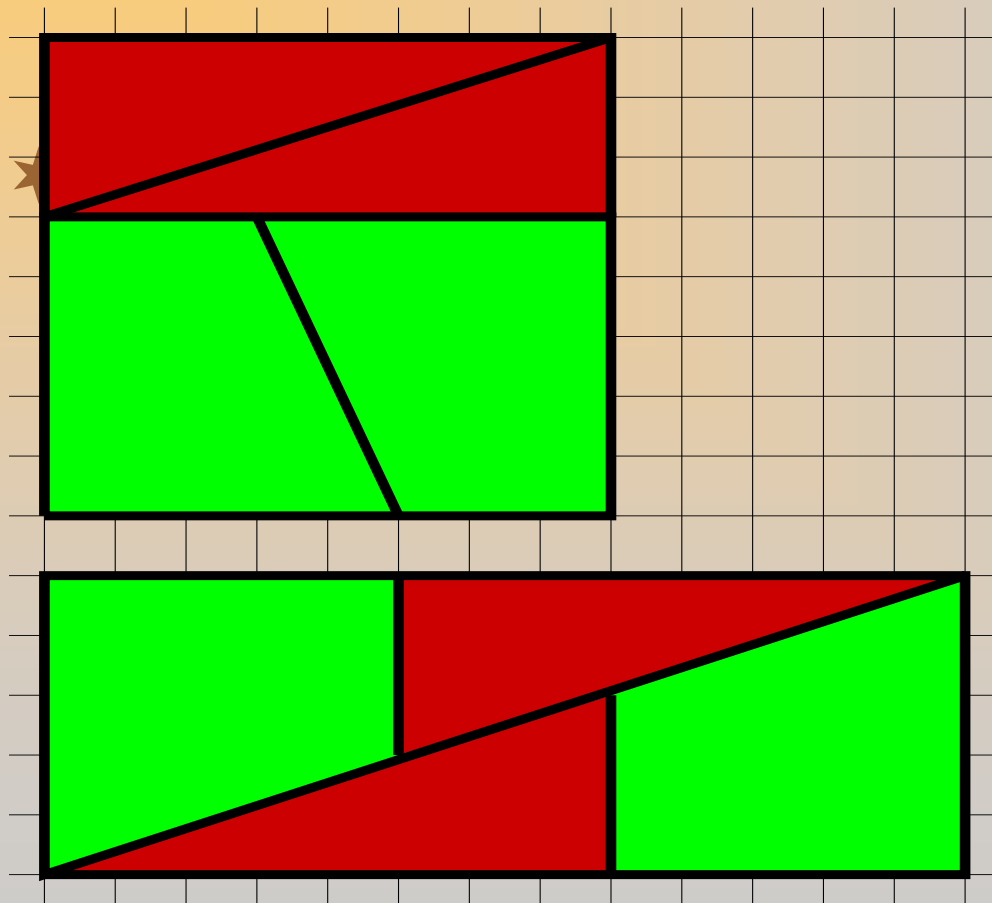
Drawings and explorations

- ★ Geometric constructions with compass and ruler
- ★ Making conclusions from a drawing
- ★ Practice with geometry improves overall mathematical and *logical* ability





Areas are equal: $64=65$





Paradoxes



- ★ Surprise and force to find an error
- ★ Develop a need for logical sequence of thoughts
- ★ Help to examine links between given information and conclusions
- ★ Force to review basic ideas and principles



Pictures and reality

Impossible figure...

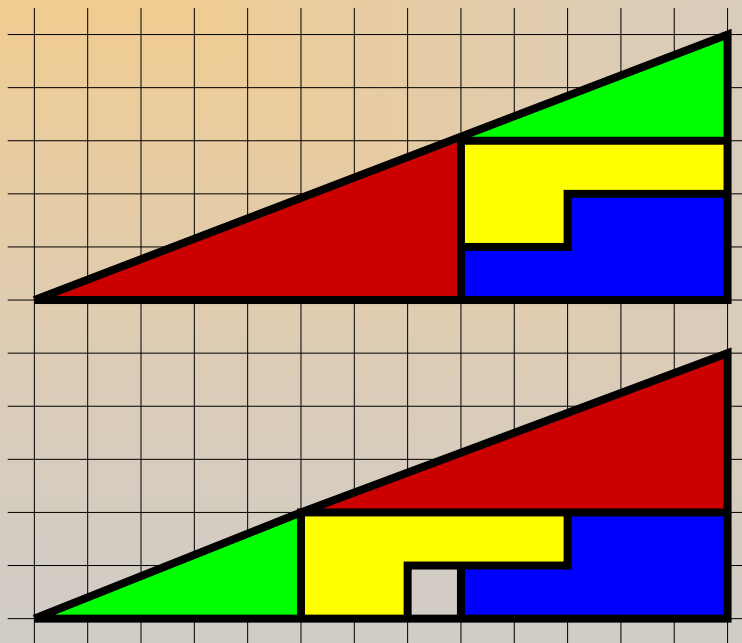




Extra square?



Explore and conclude...





Unexpected connection...

★ The sides are 1,2,3,5,8,13...

★ Fibonacci numbers

$$|F(n)F(n+1)-F(n-1)F(n+2)|=1$$

$$5*8-3*13=1$$

Slopes are close: $5/13$ and $3/8$ approx 0.38

Visual illusion!



Two sides of proofs



★ Formal proof

★ Rigor

★ Completeness

★ Books, papers

★ Symbolic

★ Definition - Statement

★ Informal proof

★ Intuition

★ Construction

★ Work in progress

★ Inactive- Iconic

★ Mental images





Conclusion:

Geometrical sophisms force the learner to

- ★ **inspect** an impossible picture,
- ★ **reveal the true structure** and properties,
- ★ **adjust the primary mental model**

and

prepares the learned to make a deductive step.





Thank you and questions

