AMAT 3260 assignment #2, due February 6

Problem 1 For the following autonomous equation answer the questions

Q1) find all equilibrium solutions and determine their types of stability;

Q2) investigate growth/decay and concavity of solutions for different initial conditions;

Q3) plot the solutions in (t, y)-plane;

Q4) give the solution by an analytic formula.

1. $y' = (1-y)(y-3)$;
2. $y' = ry - ky^2$, where parametres r and k are positive;
3. $y' = (1-y)(3-y)^2$;
4. $y' = \cos y$;
5. $y' = \cos^2 y$;
6. $y' = y - y^4$.

Problem 2 Determine whether or not each of the following equations is exact. If it is exact then find the solution.

a) $(5x+7) + (6y+11)y' = 0$;

b) $(5x+7y) + (7x+11y)y' = 0$;

c) $(e^x \sin y - 2y \sin x) + (e^x \cos y + 2\cos x)y' = 0$;

d) $y' = \dfrac{ax+by}{bx+cy}$;

e) $y' = -\dfrac{ax+by}{bx+cy}$;

f) $\dfrac{x\,dx}{(x^2+y^2)^{5/2}} + \dfrac{y\,dy}{(x^2+y^2)^{5/2}} = 0$.

Problem 3 Find value of b for which the following equation is exact and solve it.

$$(ye^{-3xy} + x^2)dx - bxe^{-3xy}dy = 0.$$

Problem 4 Solve using given integrating factor.

a) $ydx + (2x - ye^y)dy = 0, \qquad \mu(x, y) = y;$

b) $(x + 2)\sin y + (x\cos y)y' = 0, \qquad \mu(x, y) = xe^x;$

c) $(3xy + y^2) + (x^2 + xy)y' = 0, \qquad \mu(x, y) = \dfrac{1}{xy(2x + y)}.$

Problem 5 Find an integrating factor and solve.

1. $1 + \left(\dfrac{x}{y} - \sin y\right) y' = 0;$
2. $ydx + (3x - e^y)dy = 0;$
3. $xdy + (2y - e^x)dx = 0.$

Problem 6 Determine an interval in which the solution of the given initial value problem is certain to exist. Do not solve the equation.

1. $(t + 3)y' + (\ln t)y = 2t, \qquad y(1) = 2;;$
2. $y' + (\cot t)y = \cos t, \qquad y(\pi/2) = 0;$
3. $\dfrac{y'}{t} + ty = 0, \qquad y(-1) = 1.$

Problem 7 a) Find the escape velocity for a rocket launched straight upward from a point $x_0 = 0.1R$ above the surface of the earth, where $R = 4000$ miles is the radius of the earth. Neglect air resistance.

Find the initial altitude from which the rocked must be launched in order to reduce the escape velocity to 80% of its value on the earth surface.

b*) **Bonus problem**. Find the escape velocity from the moon. What do you need know about a planet (space rock) to find out how to escape from it?