Math 2051 W2008 Margo Kondratieva

Quiz 2 on Fri Feb 1: Answers

NAME and student number

1. Give a definition of *span* of n vectors.

Answer: Span of n vectors is the set of all linear combinations of the vectors.

2. Give an example of *four linearly dependent vectors* and explain, using the definition, why they are linearly dependent.

Answer: $\vec{v} = (1,0,0,0)^T$, $\vec{u} = (2,0,0,0)^T$, $\vec{w} = (0,1,2,3)^T$, $\vec{f} = (1,1,1,1)^T$. Vectors are linearly dependent if at least one of them can be written as linear combination of the rest.

Here $\vec{v} = 0.5\vec{u} + 0\vec{w} + 0\vec{f}$, which makes all four linearly dependent.

3. Is the following triple of vectors linearly dependent or linearly independent? Justify.
$\vec{u} = (1,2,0)^T$, $\vec{v} = (1,0,2)^T$, $\vec{w} = (0,1,2)^T$.

Answer: Set up matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \\ 0 & 2 & 2 \end{bmatrix}$ whose columns are the given vectors. Solve the system $AX = 0$. Since the system has only trivial solution $X = (0,0,0)^T$ we conclude that the vectors are linearly independent.

4. Find dimension and basis of the linear vector space defined as

eigenspace of A, where $A = \begin{bmatrix} 10 & 20 & 30 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$.

Answer: The only zero of the characteristic polynomial $\det(A - \lambda I)$ is $\lambda = 10$. Consider matrix $M = A - 10I = \begin{bmatrix} 0 & 20 & 30 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and solve $MX = 0$ to find the parametric solution: $x_1 = t$, $x_2 = -1.5s$, $x_3 = s$, which describes the eigenspace of A. Rewrite it as

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 0 \\ -1.5 \\ 1 \end{bmatrix}.$$

Thus, the dimension is 2 and vectors $(1,0,0)^T$ and $(0,-3,2)^T$ can be taken as a basis.

5. Find coordinates of vector $\vec{v} = (13,-20,15)^T$ relative to basis $\vec{f} = (1,-2,3)^T$, $\vec{g} = (-1,1,1)^T$.

Answer: We need to find x_1 and x_2 such that $\vec{v} = x_1\vec{f} + x_2\vec{g}$. This leads to the system of linear equations $\left[\begin{array}{cc|c} 1 & -1 & 13 \\ -2 & 1 & -20 \\ 3 & 1 & 15 \end{array}\right]$, which can be solved by elementary row operations leading to REF.

The answer is $x_1 = 7$, $x_2 = -6$.