Math 2051 W2008 Margo Kondratieva

Answers for Quiz 1 on Fri Jan 18.

NAME and student number

1. Give a definition of *null space of a matrix* A.

   Answer.
   Null space of a matrix A is a collection of all vectors X such that $AX = 0$.

2. Give an example of *linear subspace in* $\mathbf{R}^3$.

   Answer.
   Example 1: A plane $x + y - z = 0$.
   Example 2: A line through the origin in direction $(1, 2, 4)$.
   Example 3: The origin.

3. Explain using the definition whether the following is a linear subspace of $\mathbf{R}^3$:
   the plane $x - y - 2 = 0$.
   Answer.
   No. This plane does not contain the origin $x = y = z = 0$.

4. Outline the proof of Theorem 3:

Given any nonzero vector $\vec{d} = (d_1, d_2, d_3)^T$, a collection of all vectors proportional to $\vec{d}$ (that it, all vectors in the form $t\vec{d}$, where t is any real number) forms a linear vector space.

Answer.

Proof:

Consider the collection of all vectors $\vec{v}$ in the form $t\vec{d}$, where t is any real number and $\vec{d}$ is a given non-zero vector:

$$\{\vec{v} = t\vec{d} | t \in \mathbf{R}\}$$

Different values of t correspond to different elements of the collection.

1. Let $t = 0$; this gives the zero vector. Thus the zero vector is in the collection.

2. Let $\vec{v}_1 = t_1\vec{d}$ and $\vec{v}_2 = t_2\vec{d}$ be two vectors from the collection (both are proportional to $\vec{d}$). Then their sum

$$\vec{v}_1 + \vec{v}_2 = t_1\vec{d} + t_2\vec{d} = (t_1 + t_2)\vec{d} = s\vec{d}$$

is also proportional to $\vec{d}$ and thus is in the collection.

3. A multiple $k\vec{v}$ is again in the collection:

$$k\vec{v} = k(t\vec{d}) = s\vec{d}.$$

Since all three properties of a linear vector space hold, the collection is a linear vector space.

□