

# Answers

Math 2050

Test 3

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Student

Student number

1. Consider the following system of linear equations

$$\begin{cases} x + y + z = 0 \\ 2x - y - z = -3 \\ 2y + 4z = 4 \end{cases}$$

[2] (a) rewrite the system in the matrix form  $AX=B$

Answer :  $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & -1 \\ 0 & 2 & 4 \end{bmatrix}$ ,  $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 \\ -3 \\ 4 \end{bmatrix}$

[6] (b) solve the system  
 $\begin{bmatrix} 1 & 1 & 1 & | & 0 \\ 2 & -1 & -1 & | & -3 \\ 0 & 2 & 4 & | & 4 \end{bmatrix} \xrightarrow{R_2 \rightarrow R_2 - 2R_1} \begin{bmatrix} 1 & 1 & 1 & | & 0 \\ 0 & -3 & -3 & | & -3 \\ 0 & 2 & 4 & | & 4 \end{bmatrix} \xrightarrow{R_3 \rightarrow 3R_3 + 2R_2} \begin{bmatrix} 1 & 1 & 1 & | & 0 \\ 0 & -3 & -3 & | & -3 \\ 0 & 0 & 6 & | & 6 \end{bmatrix}$

$$\begin{aligned} 6z &= 6 & -3y - 3(1) &= -3 \\ z &= 1 & y &= 0 \\ && x + 1 &= 0 \\ && x &= -1 \end{aligned}$$

Answer :  $x = -1$   $y = 0$   $z = 1$

[2] (c) Complete the sentence by circling what is appropriate:

"This system of equations geometrically represents..."

- three lines intersecting at a point
- three planes intersecting at a line
- three planes intersecting at a point
- two planes intersecting with a line
- none of the above

[3] (d) express vector  $\begin{bmatrix} 0 \\ -3 \\ 4 \end{bmatrix}$  as a linear combination of the columns of matrix A from (a)

$$\begin{bmatrix} 0 \\ -3 \\ 4 \end{bmatrix} = -1 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + 0 \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + 1 \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}$$

2. Consider the following augmented matrix

$$\left[ \begin{array}{ccccc|c} x_1 & x_2 & x_3 & x_4 & x_5 & 1 \\ 1 & 0 & -3 & 4 & 5 & 1 \\ 0 & 1 & 2 & 0 & 1 & -3 \\ 0 & 0 & 0 & 2 & 0 & 4 \end{array} \right]$$

[6] (a) solve corresponding system of linear equations and write the solution in the vector form

$$2x_4 = 4$$

$$x_4 = 2$$

$$x_3 = s$$

$$x_5 = t$$

$$x_1 = -7 - 5t + 3s$$

$$x_2 = -3 - t - 2s$$

$$x_1 - 3s + 4(4)(s) + 5t = 1$$

$$\text{Answer: } \left[ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array} \right] = \left[ \begin{array}{c} -7 - 5t + 3s \\ -3 - t - 2s \\ s \\ 2 \\ t \end{array} \right] = \left[ \begin{array}{c} -7 \\ -3 \\ 0 \\ 2 \\ 0 \end{array} \right] + t \left[ \begin{array}{c} -5 \\ -1 \\ 0 \\ 0 \end{array} \right] + s \left[ \begin{array}{c} 3 \\ 1 \\ 0 \\ 0 \end{array} \right] \quad \checkmark$$

[1] (b) write the homogeneous system of equations corresponding to the system solved in (a)

$$x_1 - 3x_3 + 4x_4 + 5x_5 = 0$$

$$x_2 + 2x_3 + x_5 = 0$$

$$2x_4 = 0$$

Answer:

[3] (c) write general solution of the homogeneous system reported in (b) and explain how it is related to your answer in (a).

The general solution of the homogeneous system is the portion of the general solution of (a) that involves free variables

$$t \left[ \begin{array}{c} -5 \\ -1 \\ 0 \\ 1 \\ 0 \end{array} \right] + s \left[ \begin{array}{c} 3 \\ 1 \\ 0 \\ 0 \\ 0 \end{array} \right]$$

Answer:  $\left[ \begin{array}{c} -5 \\ -1 \\ 0 \\ 1 \\ 0 \end{array} \right] + s \left[ \begin{array}{c} 3 \\ 1 \\ 0 \\ 0 \\ 0 \end{array} \right]$

3. [8] Let  $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$ ,  $C = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$ .

Find  $A^{-1}$ ,  $C^{-1}$  and then find matrix  $X$  such that

$$AXC + A = \frac{1}{2}B^TC.$$

$$A^{-1} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$$

$$AXC = \frac{1}{2}B^TC - A$$

$$C^{-1} = \frac{1}{2} \begin{bmatrix} 2 & -4 \\ -1 & 3 \end{bmatrix}$$

$$X = A^{-1} \left( \frac{1}{2}B^TC - A \right) C^{-1}$$

$$B^T = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$

$$= \frac{1}{2} A^{-1} B^T - C^{-1}$$

$$= \frac{1}{2} \begin{pmatrix} -1 & 3 \\ 0 & -2 \end{pmatrix}$$

4. Consider augmented matrix of a system of linear equations
- $$\left[ \begin{array}{ccc|c} 1 & k & 2 & 0 \\ 0 & 3 & k+2 & 0 \\ 0 & 0 & k^2-1 & (k+1)(3k-2) \end{array} \right]$$

[3] (a) For which values of  $k$  does the system have no solutions? Explain.

The system has no solution when  $k=1$ . This makes the last equation read  $0x_3 = 2$  which can never be true.

[3] (b) For which values of  $k$  does the system have a unique solution? Explain.

The system has a unique solution when  $k \neq \pm 1$ . This means  $k^2-1 \neq 0$  and makes the final equation  $ax_3 = b$  with  $a$  and  $b$  being set values giving a unique solution.

[3] (c) For which values of  $k$  does the system have infinitely many solutions? Explain.

The system has infinitely many solutions when  $k=-1$ . This make the last equation  $0x_3 = 0$ . There are many values of  $x_3$  which make this true.

[3] (d) For which values of  $k$  does the system have zero solution? Explain.

When  $k=2/3$  the last equation reads  $((\frac{2}{3})^2 - 1)x_3 = 0$  so  $x_3$  must equal zero. By back substitution the only possible values of  $x_1$  and  $x_2$  that satisfy the system are zero.

Also ~~for  $k=-1$~~

$$\boxed{k = -1}$$

Answer:  $k = -1$

[3] (a) find  $\det A$

Answer: -16

[6] (b) find  $A^{-1}$

$$\text{Answer: } -\frac{1}{16} \begin{bmatrix} 14 & 0 & -4 \\ 15 & -3 & -3 \\ -10 & 0 & 2 \end{bmatrix}$$

[4] (c) solve  $AX = \begin{bmatrix} 1 \\ 1 \\ 8 \end{bmatrix}$

$$\text{Answer: } x = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$$

3

b)  $A = \begin{bmatrix} 14 & -15 & -10 \\ 0 & 3 & 0 \\ -4 & 3 & 2 \end{bmatrix}$   $C = \begin{bmatrix} 14 & 15 & -10 \\ 0 & -3 & 0 \\ -4 & -3 & 2 \end{bmatrix}$

$$C^\top = \begin{bmatrix} 14 & 0 & -4 \\ 15 & -3 & -3 \\ -10 & 0 & 2 \end{bmatrix}$$

c)  $X = -\frac{1}{16} \begin{bmatrix} 14 & 0 & -4 \\ 15 & -3 & -3 \\ -10 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$  ✓