MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 9 **MATH 2050 sect. 3** ANSWERS

For practice and extra credit only.

1. Use vectors addition to show that

   (a) In a regular hexagon ABCDEF, $\vec{AB} + \vec{AC} + \vec{AE} + \vec{AF} = 2\vec{AD}$
   solution: Draw a regular hexagon and mark its vertices.
   Note that $\vec{AE} = \vec{BD}$, thus $\vec{AB} + \vec{AE} = \vec{AD}$;
   similarly, $\vec{AC} = \vec{FD}$, thus $\vec{AF} + \vec{AC} = \vec{AD}$.
   In total we have $2\vec{AD}$, as required.

   (b) In a regular n-gon $A_1 \cdots A_n$ with center O and even number of vertices the sum $\sum_{k=1}^{n} \vec{OA_k} = 0$.
   solution:
   Try a square, hexagon, octagon, and observe that there are pairs of vectors which have same magnitude and opposite direction, thus they add up to zero. Same mechanism works for any even n.

2. Give a definition of

   - unit vector;
   - dot product of two vectors;
   - cross product of two vectors;
   - projection of one vector onto another vector.

3. Find the projection of vector $\vec{v} = (2, 5, 7)$ onto vector $\vec{u} = (-2, 1, 3)$, and the angle between them.

   solution:

   The dot products are: $\vec{u} \cdot \vec{v} = 22$, $\vec{u} \cdot \vec{u} = 14$, $\vec{v} \cdot \vec{v} = 78$. Thus the projection is $(11/7)\vec{u}$, and the angle is approximately 48.3 degrees.

4. A line goes via points $(1, 2, 3)$ and $(6, 5, 4)$. Another line goes through points $(3, 2, 1)$ and $(4, 5, 6)$.

   (a) Do the two lines intersect? If yes, what is the point of intersection?

   solution: Line 1 has equation $(x, y, z)^T = (6, 5, 4)^T + t(5, 3, 1)^T$.

   Line 2 has equation $(x, y, z)^T = (4, 5, 6)^T + s(1, 3, 5)^T$.

   Equating expressions for x,y,z and solving for s and t we obtain $s = -0.5$ and $t = -0.5$. Thus the lines intersect at point $x = 3.5, y = 3.5, z = 3.5$

   (b) Find the distance of each line from the origin.

Label the origin by O, and point $(1,2,3)$ from the first line by A. Then $\vec{OA} = (1,2,3)^T$. Its projection onto direction vector $(5,3,1)^T$ of line 1 is $0.4(5,3,4)^T$. The magnitude of the projection is $14/\sqrt{35}$. Magnitude of $\vec{OA}$ is $\sqrt{14}$. Thus from the right triangle with hypotenuse OA, the distance from the line 1 to the origin is $\sqrt{42/5} \approx 2.89$.

Similar calculations give the distance from the line 2 to the origin which is $\sqrt{42/5} \approx 2.89$.

5. Find equation of a plane which contains points $(1,0,1)$, $(1,2,0)$, $(0,3,1)$.

   solution:

   Label points as A,B,C. Vector $\vec{AB} = (0,2,-1)^T$, $\vec{AC} = (-1,3,0)^T$. The vector product $\vec{AB} \times \vec{AC} = (3,1,2)^T$ gives the normal vector to the plane. Thus the equation of the plane is $3(x-1) + (y-0) + 2(z-1) = 0$ or equivalently, $3x + y + 2z = 5$.

   Note that you will find the same answer by solving a system of linear equations generated by substitution of the coordinates of the points into the general equation of plane. (see assignment 3).

6. Find area of the quadrilateral with vertices $(-1,-1)$, $(2,0)$, $(3,4)$, $(-3,5)$.

   solution: Label the quadrilateral ABCD. Its area is the sum of areas of two triangles, ABC and BCD. Area of each triangle can be found via finding the side lengths and angles.

   The answer is 22.5 square units.

7. Give an example of two lines which intersect at point $(0,-1,3)$ at the right angle. Justify.

   solution:

   line 1: $(x,y,z)^T = (0,-1,3)^T + t(1,1,1)^T$

   line 2: $(x,y,z)^T = (0,-1,3)^T + s(-1,0,1)^T$

   both lines contain point $(0,-1,3)$ and their direction vectors are orthogonal (dot product is zero).

8. Give an example of a plane which is orthogonal to the plane $2x + 3y - 4z = -5$. Explain.

   solution:

   For example, $2x + z = 1$. The dot product of normal vectors of the two planes is zero. Thus they are orthogonal.