Assignment 6
MATH 2050 Sec. 3
Due: Monday Oct 29

1. For each of the following matrices find:
2. the matrix of cofactors;
3. the determinant using the cofactor (Laplace) expansion;
4. the inverse matrix using the cofactors;

$$
A=\left[\begin{array}{cc}
2 a & a^{2} \\
a^{3} & a^{4}
\end{array}\right], \quad B=\left[\begin{array}{ccc}
3 & -5 & 1 \\
5 & -10 & 5 \\
2 & 0 & -1
\end{array}\right], \quad C=\left[\begin{array}{cccc}
1 & 0 & 0 & a \\
0 & 1 & b & 0 \\
0 & c & 1 & 0 \\
d & 0 & 0 & 1
\end{array}\right]
$$

2. Find the determinant by rewriting the matrix in a triangular form

$$
C=\left[\begin{array}{ccccc}
1 & 2 & 3 & 4 & 5 \\
2 & 6 & 9 & 12 & 15 \\
1 & 2 & 6 & 8 & 10 \\
1 & 2 & 3 & 8 & 10 \\
2 & 4 & 6 & 12 & 20
\end{array}\right]
$$

3. Find conditions for $a$ and $b$ such that the matrix $\left[\begin{array}{ccc}8 & a^{2} & a \\ b & 7 & a \\ 0 & a-2 b & 0\end{array}\right]$ is invertable?
4. Let $\operatorname{det} A=3$, $\operatorname{det} B=4$, and $\operatorname{det} C=5$; and let $A$ and $B$ be $3 \times 3$ matrices, and $C$ be $4 \times 4$. Find:
$\operatorname{det}\left(A^{T} B^{2} A^{3} B^{-1}\right)$;
$\operatorname{det}\left(2 A^{2}\right)-\operatorname{det}(2 B)+\operatorname{det}(2 C)$;
$\operatorname{det}(A B C)$.
5. True of False? Explain.
a) If a matrix is symmetric $\left(A^{T}=A\right)$ then it is invertable.
b) If matrix is diagonal then it is invertable.
c) Elementary row operations always change the determinant of the matrix.
d) If $A$ and $B$ are invertable matrices then $\left((A B)^{-1}\right)^{T}=\left(A^{T}\right)^{-1}\left(B^{T}\right)^{-1}$.
