MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 6 **MATH 2050 Sec.3** ANSWERS

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1. For each of the following matrices find:

Answer

For $A = \begin{bmatrix} 2a & a^2 \\ a^3 & a^4 \end{bmatrix}$,

1. the matrix of cofactors is $\begin{bmatrix} a^4 & -a^3 \\ -a^2 & 2a \end{bmatrix}$
2. the determinant using the cofactor (Laplace) expansion is a^5.
3. the inverse matrix using the cofactors is $\frac{1}{a^5}\begin{bmatrix} a^4 & -a^2 \\ -a^3 & 2a \end{bmatrix}$.

$B = \begin{bmatrix} 3 & -5 & 1 \\ 5 & -10 & 5 \\ 2 & 0 & -1 \end{bmatrix}$,

1. the matrix of cofactors is $\begin{bmatrix} 10 & 15 & 20 \\ -5 & -5 & -10 \\ -15 & -10 & -5 \end{bmatrix}$
2. the determinant using the cofactor (Laplace) expansion is -25.
3. the inverse matrix using the cofactors is $B^{-1} = \frac{1}{5}\begin{bmatrix} -2 & 1 & 3 \\ -3 & 1 & 2 \\ -4 & 2 & 1 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & b & 0 \\ 0 & c & 1 & 0 \\ d & 0 & 0 & 1 \end{bmatrix}$.

1. the matrix of cofactors is $\tilde{C} = \begin{bmatrix} 1-bc & 0 & 0 & -d(1-bc) \\ 0 & 1-ad & -c(1-ad) & 0 \\ 0 & -b(1-ad) & 1-ad & 0 \\ -a(1-bc) & 0 & 0 & 1-bc \end{bmatrix}$.
2. the determinant using the cofactor (Laplace) expansion is $\det C = (1-bc)(1-ad)$
3. the inverse matrix using the cofactors is

$$C^{-1} = \frac{1}{(1-bc)(1-ad)}\begin{bmatrix} 1-bc & 0 & 0 & -a(1-bc) \\ 0 & 1-ad & -b(1-ad) & 0 \\ 0 & -c(1-ad) & 1-ad & 0 \\ -d(1-bc) & 0 & 0 & 1-bc \end{bmatrix}.$$

2. Find the determinant by rewriting the matrix in a triangular form

$$C = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 6 & 9 & 12 & 15 \\ 1 & 2 & 6 & 8 & 10 \\ 1 & 2 & 3 & 8 & 10 \\ 2 & 4 & 6 & 12 & 20 \end{bmatrix}.$$

Solution: after performing EROs: R2=R2-2R1, R3=R3-R1, R4=R4-R1, R5=R5-2R1, R5=R5-R4, we obtain a the upper-triangular matrix. The product of its diagonal elements gives the determinant: $1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$.

3. Find conditions for a and b such that the matrix $\begin{bmatrix} 8 & a^2 & a \\ b & 7 & a \\ 0 & a-2b & 0 \end{bmatrix}$ is invertable?

Solution: A matrix is invertable if its determinant is not zero. $detA = (a-2b)(b-8)a$. Thus unless either $a = 2b$, or $b = 8$, or $a = 0$, the matrix is invertable.

4. Let $\det A = 3$, $\det B = 4$, and $\det C = 5$; and let A and B be 3×3 matrices , and C be 4×4. Find:

   Solution:

   $\det(A^T B^2 A^3 B^{-1}) = 3^4 \cdot 4 = 324$;

   $\det(2A^2) - \det(2B) + \det(2C) = 2^3 \cdot 3^2 - 2^3 \cdot 4 + 2^4 \cdot 5 = 120$;

   $\det(ABC)$ = nothing!!!. Because the product of matrices with given dimentions is not possible.

5. True of False? Explain.

   a) If a matrix is symmetric ($A^T = A$) then it is invertable.

   False. $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ is symmetric and not invertable.

   b) If matrix is diagonal then it is invertable.

   False. Some diagonal elements can be zero.

   c) Elementary row operations always change the determinant of the matrix.

   False. ERO "Replace a row with itself plus multiple of another row" does not change the determinant.

   d) If A and B are invertable matrices then $((AB)^{-1})^T = (A^T)^{-1}(B^T)^{-1}$.

   True. $((AB)^{-1})^T = (B^{-1}A^{-1})^T = (A^{-1})^T(B^{-1})^T = (A^T)^{-1}(B^T)^{-1}$

   we use the properties $(AB)^T = B^T A^T$, $(AB)^{-1} = B^{-1}A^{-1}$, $(A^{-1})^T = (A^T)^{-1}$.