MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 5 **MATH 2050** ANSWERS

1. Find the inverse of each of the following matrices or explain why it is not possible.

   $A = \begin{bmatrix} 10 & 20 \\ 2 & 4 \end{bmatrix}$. $\det A = 0$ thus the inverse matrix does not exist.

   $B = \begin{bmatrix} 10 & 20 \\ 5 & 4 \end{bmatrix}$. $\det B = -60$; $B^{-1} = -\frac{1}{60}\begin{bmatrix} 4 & -20 \\ -5 & 10 \end{bmatrix}$

   $C = \begin{bmatrix} 3 & -5 & 1 \\ 5 & -10 & 5 \\ 2 & 0 & -1 \end{bmatrix}$, $C^{-1} = \frac{1}{5}\begin{bmatrix} -2 & 1 & 3 \\ -3 & 1 & 2 \\ -4 & 2 & 1 \end{bmatrix}$.

   $D = \begin{bmatrix} -2 & 1 & 3 \\ -3 & 1 & 2 \\ -4 & 2 & 1 \end{bmatrix}$, $D^{-1} = \frac{1}{5}\begin{bmatrix} 3 & -5 & 1 \\ 5 & -10 & 5 \\ 2 & 0 & -1 \end{bmatrix}$.

   $F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 2 & 2 \end{bmatrix}$, This is not a square matrix thus it is not invertable.

   $G = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 3 & 3 & 3 \end{bmatrix}$. This matrix is not invertable.

2. Solve the system of equations by writing it in the form $AX = B$ and finding A^{-1}. Check your answer.

   Hint: you can use your result obtained in problem 1 if appropriate.

   (a) $\begin{cases} 2x - 5y = -14 \\ -x + 2y = 5 \end{cases}$

   $A = \begin{bmatrix} 2 & -5 \\ -1 & 2 \end{bmatrix}$; $A^{-1} = (-1)\begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix}$; $\begin{bmatrix} x \\ y \end{bmatrix} = (-1)\begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} -14 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$.

   Answer: $x = 3, y = 4$

   (b) $\begin{cases} -2x + 13y = 24 \\ 11x + 23y = 35 \end{cases}$

   Answer: $x = -1, y = 2$

   (c) $\begin{cases} -2x + y + 3z = -1 \\ -3x + y + 2z = -3 \\ -4x + 2y + 1z = -2 \end{cases}$

   Answer: $x = 2, y = 3, z = 0$

   (d) $\begin{cases} -2x + y + 3z = 1 \\ -3x + y + 2z = -1 \\ -4x + 2y + 1z = -3 \end{cases}$

   Answer: $x = 1, y = 0, z = 1$

3. Explain why each of the following matrices is elementary. Find its inverse in the easy way.

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2^k & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, B^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2^{-k} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -5 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}. C^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

4. Explain in your words, using the definition of inverse matrix and/or examples of your choice, why each of the following statements is true.

   a) If a matrix has an inverse then it must be a square matrix.

   the only way to respect the dimentions in multiplication AA^{-1} and $A^{-1}A$ and receive the same identity matrix I is to have both A and A^{-1} square matrices of the same size.

   b) Not any square matrix is invertable.

   Matrices A and G in problem 1 are square but not invertable; their determinant is zero.

   c) Every elementary matrix is invertable.

   One can always undo the "do".

   d) If matrix A is invertable then the system $AX = B$ has a unique solution for any vector-column B.

   this solution is $X = A^{-1}B$ (as illustrated in problem 2).