MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 3 **MATH 2050** DUE: THUR. MAY 31.

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1. For each system of linear equations given below identify the rank of the matrix of coefficients. Find the solution (if it exists). In each solvable case observe the relation between the rank, number of parameters and total number of unknowns.

(a) $\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right]$

(b) $\left[\begin{array}{cccc|c} 0 & 0 & 1 & 1 & 2 \\ 2 & 1 & 0 & 0 & 3 \\ 0 & 0 & 3 & 3 & 10 \end{array}\right]$

(c) $\left[\begin{array}{cccc|c} 2 & 2 & 2 & 2 & 3 \\ 0 & 4 & 4 & 4 & 3 \\ 0 & 0 & 1 & 1 & 3 \end{array}\right]$

(d) $\left[\begin{array}{cccc|c} 2 & 2 & 2 & 2 & 4 \\ 4 & 4 & 4 & 4 & 8 \\ 8 & 8 & 8 & 8 & 16 \end{array}\right]$

2. Find an equation of the curve or surface passing through given points:

   (a) line $ax + by = c$ through points $(2, 1)$ and $(6, 6)$;

   (b) plane $ax + by + cz + d = 0$ through $(1, 1, 1)$, $(6, 2, 0)$, $(-1, 0, 1)$

   (c) circle $x^2 + y^2 + ax + by + c = 0$ through $(5, 1)$, $(8, -2)$, $(5, -5)$.

   (d) parabola $ax^2 + bx + cy + d = 0$ through $(-3, -3)$, $(-1, -7)$, $(1, -19)$, $(-2, -4)$.

3. Find value a such that the system has non-trivial solutions. Find the solutions.

(a) $\begin{cases} x + y - z = 0 \\ ay - z = 0 \\ x + y + az = 0 \end{cases}$

(b) $\begin{cases} x + y + az = 0 \\ ax + y + z = 0 \\ x + y - z = 0 \end{cases}$

4. True or False?

   (a) Not every homogeneous system of linear equations has a solution.

   (b) If there exists a trivial (zero) solution then the system is homogeneous.

   (c) If the system is not inconsistent then the rank of the matrix of coefficients must be equal to the number of variables.

   (d) The number of parameters is equal to the number of leading ones in the REF.

5. Compose your own True-or-False question. Answer is.