1. Give an example of a system of linear equations with four variables and the rank as specified. Give a solution for your system and prove by substitution that your solution works for your system. Observe the relation between the rank, number of parameters and total number of the unknowns (variables).
(a) rank is 2 .
(c) rank is 3 .
(b) rank is 1 .
(d) rank is 4 .
2. Find an equation of the plane passing through given points. Is the plane unique?
(a) plane $a x+b y+c z+d=0$ through $(1,1,0),(5,0,1),(2,2,1)$
(b) plane $a x+b y+c z+d=0$ through ( $0,1,1$ ), (1,2,2), (2,3,3)
3. Find an equation of the curve passing through given points if it exists. Otherwise elain why it does not exist.
(a) line $a x+b y=c$ through points $(2,1),(1,2)$, and $(6,6)$;
(b) circle $x^{2}+y^{2}+a x+b y+c=0$ through $(7,-5),(5,-7),(5,-3)$.
(c) circle $x^{2}+y^{2}+a x+b y+c=0$ through $(0,1),(1,2),(2,3)$.
(d) parabola $a x^{2}+b x+c y+d=0$ through (1,4), (4, 13), ( $-2,1$ ), $(-5,4)$.
4. Find value $a$ such that the system has non-trivial solutions. Find the solutions.
(a) $\left\{\begin{array}{l}2 x+6 y+12 z=0 \\ x+2 y+z=0 \\ 2 x+3 y+a z=0\end{array}\right.$
(b) $\left\{\begin{array}{l}x-6 y+5 z=0 \\ x+a y-3 z=0 \\ -x+2 y-z=0\end{array}\right.$
5. True or False? Explain.
(a) Every homogeneous system of linear equations has a parametric solution.
(b) Rank of a system of linear equations is always less than the number of variables.
(c) If a system is inconsistent then it is definedly not homogeneous.
(d) There is a circle through any three point in the $X Y$-plane.
6. Compose your own True-or-False question. Answer is.
