1. For each system of linear equations given below identify the rank of the matrix of coefficients. Find the solution (if it exists). In each solvable case observe the relation between the rank, number of parameters and total number of unknowns.
(a) $\left[\begin{array}{llll|l}1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 2 & 1 & 4\end{array}\right]$
(c) $\left[\begin{array}{llll|l}2 & 2 & 2 & 2 & 3 \\ 4 & 4 & 4 & 4 & 3 \\ 1 & 1 & 1 & 1 & 3\end{array}\right]$
(b) $\left[\begin{array}{llll|c}0 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 2 & 3 & 10\end{array}\right]$
(d) $\left[\begin{array}{llll|l}2 & 2 & 2 & 2 & 4 \\ 4 & 4 & 4 & 4 & 8 \\ 1 & 1 & 1 & 1 & 2\end{array}\right]$
2. Find an equation of the curve or surface passing through given points:
(a) line $a x+b y=c$ through points $(1,2)$ and $(-100,200)$;
(b) plane $a x+b y+c z+d=0$ through $(0,1,20),(1,20,0),(1,0,0)$
(c) circle $x^{2}+y^{2}+a x+b y+c=0$ through $(-2,1),(5,0),(4,1)$.
(d) parabola $x^{2}+a x+b y+c=0$ through $(0,9),(4,1),(2,3)$.
3. Find value $a$ such that the system has non-trivial solutions. Find the solutions.
(a) $\left\{\begin{array}{l}x-2 y+z=0 \\ x+a y-3 z=0 \\ -x+6 y-5 z=0\end{array}\right.$
(b) $\left\{\begin{array}{l}x+2 y+z=0 \\ x+3 y+6 z=0 \\ 2 x+3 y+a z=0\end{array}\right.$
4. True or False?
(a) Every system of linear equations has a solution.
(b) Every homogeneous system of linear equations has a non-trivial solution.
(c) If a system of linear equations has zero solution then it must be homogeneous.
(d) If in a system of linear equations the number of variables is less then the number of equations then the system has no solutions.
5. Compose your own True-or-False question. Answer is.
