MEMORIAL UNIVERSITY OF NEWFOUNDLAND DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 2050

Answers

- 1. For each system of linear equations given below identify the rank of the matrix of coefficients. Find the solution (if it exists). In each solvable case observe the relation between the rank, number of parameters and total number of unknowns.
 - (a) $\begin{bmatrix} 1 & 2 & 3 & 4 & | & 5 \\ 0 & 1 & 2 & 3 & | & 4 \\ 0 & 0 & 1 & 2 & | & 3 \\ 0 & 0 & 0 & 1 & | & 2 \end{bmatrix}$

Assignment 3

Answer: This system is in REF. Rank is 4. Number of unknowns is 4. The system has unique solution x = 0, y = 0, z = -1, w = 2.

(b) $\begin{bmatrix} 0 & 0 & 1 & 1 & 2 \\ 2 & 1 & 0 & 0 & 3 \\ 0 & 0 & 3 & 3 & 10 \end{bmatrix}$

Answer: Transform the system in REF. Rank is 2. Number of unknowns is 4. The last equation reads: 0 = 4 thus the system is inconsistent: the system has no solutions.

(c) $\begin{bmatrix} 2 & 2 & 2 & 2 & 3 \\ 0 & 4 & 4 & 4 & 3 \\ 0 & 0 & 1 & 1 & 3 \end{bmatrix}$

Answer: Transform the system in REF. Rank is 3. Number of unknowns is 4. The system has a parametric solution (one parameter)

$$w = t$$
, $z = 3 - t$, $y = -\frac{9}{4}$, $x = \frac{3}{4}$

(d) $\begin{bmatrix} 2 & 2 & 2 & 2 & 4 \\ 4 & 4 & 4 & 4 & 8 \\ 8 & 8 & 8 & 8 & 16 \end{bmatrix}$

Answer: Rank is 1. Number of unknowns is 4. The system has a parametric solution (three parameters)

 $w=t, \quad z=s, \quad y=q \quad x=2-t-s-q.$

- 2. Find an equation of the curve or surface passing through given points:
 - (a) line ax + by = c through points (2, 1) and (6, 6); Answer: Substitute coordinates of the points into equation of line to get the system of linear equations for coefficients a, b, c.
 - (b) $\begin{cases} 2a+b-c=0\\ 6a+6b-c=0 \end{cases}$ This system has parametric solution c = 3t, b = -2t, a = 5t/2,where t is any number. Pick t = 2, then the equation of the line is 5x - 4y = 6. Observe that this is not a unique way to write the equation (since one can multiply the whole equation by any nonzero number). The line hovewer is unique.

(c) plane ax + by + cz + d = 0 through (1, 1, 1), (6, 2, 0), (-1, 0, 1)

Answer: Substitute coordinates of the points into equation of planee to get the system of linear equations for coefficients a, b, c, d.

$$a+b+c+d=0$$

(d) $\begin{cases} 6a + 2b + d = 0 & \text{This sytem has a parametric solution. The equation of the} \\ -a + c + d = 0 & \text{plane is } x - 2y + 3z - 2 = 0. & \text{Observe that this is not a unique way to write the} \end{cases}$

plane is x - 2y + 3z - 2 = 0. Observe that this is not a unique way to write the equation (since one can multiply the whole equation by any nonzero number). The plane hovewer is unique.

(e) circle $x^2 + y^2 + ax + by + c = 0$ through (5, 1), (8, -2), (5, -5). Answer: Substitute coordinates of the points into equation of circle to get the system of linear equations for coefficients a, b, c.

$$\int 5a + b + c = -26$$

(f) $\begin{cases} 8a - 2b + c = -68 & \text{This sytem has unique solution } c = 20, b = 4, a = -10. \text{ Then} \\ 5a - 5b + c = -50 \end{cases}$

the equation of the circle is $x^2 + y^2 - 10x + 4y + 20 = 0$. One can complete the squares to get $(x - 5)^2 + (y + 2)^2 = 9$. Thus the circle has radius 3 and center at (5, -2).

- (g) parabola $ax^2 + bx + cy + d = 0$ through (-3, -3), (-1, -7), (1, -19), (-2, -4). Answer: Substitute coordinates of the points into equation of parabola to get a system of linear equations for coefficients a, b, c, d. The equation of the parabola is $x^2 + 6x + y + 12 = 0$.
- 3. Find value a such that the system has non-trivial solutions. Find the solutions.

(a)
$$\begin{cases} x+y-z=0\\ ay-z=0\\ x+y+az=0 \end{cases}$$

Answer: Transform the system in the REF. $\begin{bmatrix} 1 & 1 & -1 & | & 0 \\ 0 & a & -1 & | & 0 \\ 0 & 0 & a+1 & | & 0 \end{bmatrix}.$

Note that if a+1 = 0 then the last equation becomes 0 = 0 and system has parametric solution z = t, y = -t, x = 2t.

Note that if a = 0 then both the second and the third equations become z = 0. Thus system has parametric solution z = 0, y = -t, x = t.

(b)
$$\begin{cases} x + y + az = 0 \\ ax + y + z = 0 \\ x + y - z = 0 \end{cases}$$

Answer: Transform the system in REF $\begin{bmatrix} 1 & 1 & a & | & 0 \\ 0 & 1-a & 1-a^2 & | & 0 \\ 0 & 0 & a+1 & | & 0 \end{bmatrix}$.

We have:

If a = 1 then x = -t, y = t, z = 0. If a = -1 then x = t, y = 0, z = t.

- 4. True or False?
 - (a) Not every homogeneous system of linear equations has a solution. Answer: False.
 - (b) If there exists a trivial (zero) solution then the system is homogeneous. Answer: **True**.
 - (c) If the system is not inconsistent then the rank of the matrix of coefficients must be equal to the number of variables.
 Answer: False.
 - (d) The number of parameters is equal to the number of leading ones in the REF. Answer: False.
- 5. Compose your own True-or-False question. Answer is.