1. There are geese and pigs in a farm. They collectively have 20 legs. How many geese and pigs are in the farm? Is the answer unique? How many answers are possible? How do you know that you listed them all?
2. Give an example of an augmented matrix of a system of linear equations with its matrix of coefficients in the row-echelon form (REF) such that:
a) the system has a unique solution;
b) the system has infinitely many solutions;
c) the system has no solutions;
3. Solve the given systems by reduction corresponding Augmented Matrix to Row-Echelon Form (REF). Find the rank of the matrix of coefficients.
(a) $\left\{\begin{array}{l}2 x+3 y+z=1 \\ x+y+z=3 \\ 3 x+4 y+2 z=4\end{array}\right.$
(c) $\left\{\begin{array}{l}-x+2 y-z=2 \\ -2 x+2 y+z=4 \\ 3 x+2 y+2 z=5 \\ -3 x+8 y+5 z=17\end{array}\right.$
(b) $\left\{\begin{array}{l}x-y+2 z=4 \\ 2 x+3 y-z=1 \\ 7 x+3 y+4 z=7\end{array}\right.$
4. In each of the following find conditions for $a, b, c$ such that the system has no solutions, a unique solution, or infinitely many solutions.
(a) $\left\{\begin{array}{l}x-y+2 z=a \\ 2 x+3 y-z=b \\ 7 x+3 y+4 z=c\end{array}\right.$
(c) $\left\{\begin{array}{l}x-y+2 z=1 \\ 2 x+3 y-z=1 \\ 7 x+3 y+b z=5\end{array}\right.$
(b) $\left\{\begin{array}{l}x-y+2 z=1 \\ 2 x+3 y-z=2 \\ 7 x+3 y+a z=1\end{array}\right.$
5. Find all solutions to the following system in parametric form in two ways. Use sample value of parameter to obtain a particular numeric solution from one of the forms. Then find value of parameter in another form that yield the same numeric solution.
$\left\{\begin{array}{l}x+2 y-3 z=4 \\ 3 x-2 y=6\end{array}\right.$
