MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 2 **MATH 2050** DUE: THUR MAY 24

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1. Margo has 1.05 dollars in dimes nickels, and pennies. If there are 17 coins in all, how many coins of each type can she have?

   Answer: Let d be the number of dimes, n - number of nickels, and p - number of pennies that Margo has. Then we have the following system of linear equations

   $$p + n + d = 17, \quad p + 5n + 10d = 105$$

   with the restriction that the solutions must be all non-negative integers. There are two sets of such solutions: $p = 5$, $n = 4$, $d = 8$ and $p = 0$, $n = 13$, $d = 4$. Thus the answer is: Margo has either 8 dimes 4 nickels, and 5 pennies or 4 dimes 13 nickels, and no pennies.

2. Solve the given systems by reduction corresponding Augmented Matrix to Row-Echelon Form (REF). Find the rank of the matrix of coefficients.

   (a) $\begin{cases} x + y + z = 2 \\ x + z = 1 \\ 2x + 5y + 2z = 7 \end{cases}$

   Answer: The system in REF is $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$; rank is 2; The solution

   $$x = 1 - t, \quad y = 1, \quad z = t,$$

   where t is any number.

   (b) $\begin{cases} 5x + y = 2 \\ 3x - y + 2z = 1 \\ x + y - z = 6 \end{cases}$

   Answer: REF is $\left[\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 0 & 1 & -5/4 & 17/4 \\ 0 & 0 & 0 & 11 \end{array}\right]$. The rank is 2; the system is inconsistent (last equation reads $0 = 11$). There are no solutions.

   (c) $\begin{cases} x + y + 2z = 8 \\ x - 3y - 4z = 4 \\ 3x - y + z = 0 \end{cases}$

   Answer: From REF we conclude that Rank is 3. The system has unique solution $x = 17$, $y = 31$, $z = -20$.

3. In each of the following find conditions for a, b, c such that the system has no solutions, a unique solution, or infinitly many solutions.

(a) $\begin{cases} x - y + 2z = a \\ 3x + y - z = b \\ 5x + 3y - 4z = c \end{cases}$

Answer: When the system is written in the REF the last equation reads: $0 = a - 2b + c$. Thus for $a - 2b + c \neq 0$ there are no solutions. For $a - 2b + c = 0$ there are infinitly many solutions. A unique solution is not possible for that system.

(b) $\begin{cases} x + ay = 0 \\ y + bz = 0 \\ z + cx = 0 \end{cases}$

Answer: When the system is written in REF the last equation reads: $(abc + 1)z = 0$. Thus if $abc \neq -1$ the system has a unique solution $x = y = z = 0$. If $abc = -1$ the system has infinitly many solutions. Since the system is homogeneous is is impossible for it to have no solutions.

(c) $\begin{cases} 3x - y + 2z = 3 \\ x + y - z = 2 \\ 2x - 2y + 3z = b \end{cases}$

Answer: From REF we conclude that when $b \neq 1$ there are no solutions; when $b = 1$ there are infinitly many solutions. A unique solution is not possible for that system.

4. Find all solutions to the following system in parametric form in two ways. Use sample value of parameter to obtain a particular numeric solution from one of the forms. Then find value of parameter in another form that yield the same numeric solution.

$\begin{cases} x + y + z = 2 \\ x + y - z = 3 \end{cases}$

Answer: By adding and subtracting the equations, the system can be rewritten as $\begin{cases} x + y = 5/2 \\ z = -1/2 \end{cases}$ Then the first way to write its parametric solution is $x = t$, $y = 5/2 - t$, $z = -1/2$. Another way is $x = 5/2 - s$, $y = s$, $z = -1/2$. Then point $x = 0$, $y = 5/2$, $z = -1/2$ corresponds to the choice $t = 0$ or $s = 5/2$.

5. (Partial fraction decomposition): Find a, b, c such that

$$\frac{9x^2 + 4x + 7}{(x^2 + 2)(2x - 1)} = \frac{ax + b}{x^2 + 2} + \frac{c}{2x - 1}.$$

Answer: Add two fractions in the right hand side to have a common denominator and set the coefficients of x^2, x and free terms of the nominators equal. This leads to the system $\begin{cases} 2a + c = 9 \\ 2b - a = 4 \\ 2c - b = 7 \end{cases}$ Solving the system we find $a = 2$, $b = 3$, $c = 5$.