MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 2 **MATH 2050 sect. 3** DUE: FRIDAY SEPT 22

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1. Find all solutions to the following in parametric form in two ways. Use sample value(s) of parameter(s) to obtain a particular numeric solution from one of the forms. Then find value(s) of parameter(s) in another form that yield the same numeric solution.

   (a) $4x - y = 2$

   Solution

   First way: $y = t$, $x = 1/2 + t/4$, t is a real number;

   Second way: $x = s$, $y = 4s - 2$, s is a real number;

   Let $t = 0$. Thet first parametric form gives point $(1/2, 0)$. This point corresponds to $s = 1/2$ in the second parametric form.

   (b) $\begin{cases} x + y + z = 2 \\ x - y - z = 3 \end{cases}$

   Solution

   By adding two equations we get $2x = 5$; By subtracting them we get $2y + 2z = -1$. Thus parametric solution is:

   First way: $x = 5/2$, $y = t$, $z = -1/2 - t$, t is a real number;

   Second way: $x = 5/2$, $y = -1/2 - s$, $z = s$, s is a real number;

   Let $t = 0$. Thet first parametric form gives point $(5/2, 0, -1/2)$. This point corresponds to $s = -1/2$ in the second parametric form.

2. Show that the system of 3 equations $x + 2y - z = a$, $2x + y + 3z = b$, $x - 4y + 9z = c$ has no solutions unless $c = 2b - 3a$. In the latter case, how many solutions does the system have?

   Solution In the augmented matrix form the system can be written as $\left[\begin{array}{ccc|c} 1 & 2 & -1 & a \\ 2 & 1 & 3 & b \\ 1 & -4 & 9 & c \end{array}\right]$.

   Reduce it by EROs $R_2 := R_2 - 2R_1$, $R_3 := R_3 - R_1$, $R_3 := R_3 - 2R_2$ to REF

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & a \\ 0 & -3 & 5 & b - 2a \\ 0 & 0 & 0 & c + 3a - 2b \end{array}\right].$$

   From the last row we conclude that the system has solutions only if $c + 3b - 2a = 0$.

   In this case it will have an infinitly many solutions (each for each value of parameter t) $z = t$, $y = (-b + 2a + 5t)/3$, $x = -3a + 2b/3 - 7t/3$.

3. (Partial fraction decomposition): Find a, b, c such that

$$\frac{x^2 - x + 3}{(x^2 + 2)(2x - 1)} = \frac{ax + b}{x^2 + 2} + \frac{c}{2x - 1}.$$

(see hint in Text, Q.1.1.15)

Solution

$$x^2 - x + 3 = (ax + b)(2x - 1) + c(x^2 + 2) \quad \to \quad x^2 - x + 3 = (2a + c)x^2 + (2b - a)x + 2c - b$$

Equating coefficients of x^2, x and the constant terms we get a system of linear equations for a, b, c: $2a + c = 1$, $2b - a = -1$, $2c - b = 3$. Solving the system we get $a = -1/9$, $b = -5/9$, $c = 11/9$.

4. Solve the given systems by reduction corresponding Augmented Matrix to Reduced Row-Echelon Form (REF).

(a) $\begin{cases} x + y + 2z = -1 \\ 2x + y + 3z = 0 \\ -2y + z = 2 \end{cases}$

Solution

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 2 & 1 & 3 & 0 \\ 0 & -2 & 1 & 2 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5/3 \\ 0 & 1 & 0 & -4/3 \\ 0 & 0 & 1 & -2/3 \end{array}\right]$$

Thus the answer is $x = 5/3$, $y = -4/3$, $z = -2/3$.

(b) $\begin{cases} 5x + y = 2 \\ 3x - y + 2z = 1 \\ x + y - z = 5 \end{cases}$

Solution

$$\left[\begin{array}{ccc|c} 5 & 1 & 0 & 2 \\ 3 & -1 & 2 & 1 \\ 1 & 1 & -1 & 5 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & -1 & 5 \\ 0 & -4 & 5 & -14 \\ 0 & 0 & 0 & 9 \end{array}\right]$$

The last equation $0 = 9$ can't be satisfied. Thus there are no solutions.

5. Carry each of the following matrices to Reduced REF

(a) $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -1 & 2 & 1 & 2 & 1 & -1 \\ 0 & 1 & -2 & 2 & 7 & 2 & 4 \\ 0 & -2 & 4 & 3 & 7 & 1 & 0 \\ 0 & 3 & -6 & 1 & 6 & 4 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 1/2 & -1/2 & 3/2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & -2 & 3 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(f) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$