MEMORIAL UNIVERSITY OF NEWFOUNDLAND DEPARTMENT OF MATHEMATICS AND STATISTICS

Assignment 1 MATH 2050 Answers

- 1. Find all solutions for the following linear equation by writing the solution in parametric form. How many parameters are in the solution?
 - (a) 27x + 54y = 81

Answer: The equation is equivalent to x = -2y + 3; Let y = t, where t is any number (t is a parameter); then x = -2t + 3.

All (infinitly many) solutions can be found this way by picking various values for t. There is one parameter in the solution.

(b) 27x + 81 = 0

Answer: Here x = 3. There are no parameters in the solution; the solution is unique.

(c) x - 2y + 3z - 4w + 5v - 6u = 7Answer: Here values of five variables can be chosen arbitrary and independently. Thus there are five parameters. The sixth variable then will be defined via them. E.g.

$$y = t, z = s, w = p, v = q, u = r, x = 7 + 2t - 3s + 4p - 5q + 6r,$$

where t, s, p, q, r are any numbers.

(d) $\sum_{k=1}^{n} kx_k = n^2$, where $n \ge 2$.

Answer: Here values of n-1 variables can be chosen arbitrary and independently. Thus there are n-1 parameters in the solution. Then the n-th variable will be defined via them. E.g.

$$x_2 = t_2, x_3 = t_3, ..., x_n = t_n, \quad x_1 = n^2 - \sum_{k=2}^n k t_k.$$

- 2. Solve each of the systems algebraically and geometrically (or argue that it does not have a solution). Write the augmented matrix corresponding to each of the systems.
 - (a) x 3y = 5 2x + y = 1Answer: x = 8/7, y = -9/7; two lines intersect.
 - (b) x 2y = 6y + 1 = 0

x - 4 = 0

Answer: x = 4, y = -1; all three lines intersect at one point;

(c) 2x + y = 3

6x + 3y = 910 - 4y = 8x

Answer: no solutions; the second and third equations represent two parallel lines, so the three lines do not intersect at one point.

(d) 2x + 4y = 610y + 5x = 15

Answer: infinitly many solutions (parametric solution) x = 3 - 2t, y = t, t is any number. The two lines coinside.

3. Write a linear system corresponding to the given augmented matrix.

 $\begin{bmatrix} -5 & 2 & -3 & 4 & | & 1 \\ 0 & -10 & 0 & 1 & | & -100 \end{bmatrix}$ Answer: -5x + 2y - 3z + 4w = 1, -10y + w = -100.

4. Give an example of a system of four linear equations in two variables that has a unique solution.

Answer: x + 1 = 0, y - 1 = 0, x + y = 0, 2x + 3y = 1.

5. Margo needs 200mg of vitamin A, 100mg of vitamin D, and 140mg of vitamin E per week. She has three supplements: the first contains 20% vitamin A, 20% vitamin D and 20% vitamin E; the second contains 10% vitamin A, 30% vitamin D and 40% vitamin E; the third contains 50% vitamin A, 10% vitamin D and 20% vitamin E. How much of each supplement should she eat each week?

Answer: Let x be the amount (in mg) of supplement 1, y - of supplement 2, z - of supplement 3 that Margo needs to eat. Then the system of linear equations is as follows:

$$0.2x + 0.1y + 0.5z = 200, \quad 0.2x + 0.3y + 0.1z = 100, \quad 0.2x + 0.4y + 0.2z = 140$$

Its solution is x = 200, y = 100, z = 300. Thus the answer is: Margo needs to eat 200mg of supplement 1, 100mg of supplement 2, and 300mg of supplement 3.

6. Compose your own word problem that requires solution of a system of linear equations. Solve the problem.