This assignment is a collection of problems from the last part of the course. It covers material you must know for the final examination, but is **not** to be turned in for credit. Solutions will be posted during the last week of the semester.

1. Consider the system of equations $\begin{aligned} x_2 - x_3 &= 8 \\ x_1 + 2x_2 + x_3 &= 5 \\ x_1 + x_3 &= -7. \end{aligned}$

   i. Write this in the form $A\mathsf{x} = \mathsf{b}$.

   ii. Solve the system by finding A^{-1}.

   iii. Express $\begin{bmatrix} 8 \\ 5 \\ -7 \end{bmatrix}$ as a linear combination of the vectors $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$.

2. Express $\begin{bmatrix} 0 & -2 & 1 \\ 0 & 1 & 0 \\ 1 & -5 & 2 \end{bmatrix}$ as the product of elementary matrices.

3. Let $A = \begin{bmatrix} -1 & 2 & 4 \\ 0 & 3 & 5 \\ 2 & -2 & 3 \end{bmatrix}$.

   (a) Find the matrix M of minors, find the matrix C of cofactors and compute the product AC^T.

   (b) Find $\det A$.

   (c) If A is invertible, find A^{-1}.

4. Suppose A is a 2×2 matrix with $\det A = 5$ and cofactor matrix $C = \begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$.

   What is A?

5. Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 1 \\ 2 & -1 & 3 \end{bmatrix}$.

   Find the determinant of A expanding by cofactors of the second column.

6. If A and B are 3×3 matrices, $\det A = 2$ and $\det B = -5$, find $\det A^T B^{-1} A^3 (-B)$.

7. Let $B = \begin{bmatrix} 1 & 2 & 1 \\ 3 & -2 & 0 \\ -1 & 4 & 1 \end{bmatrix}$.

   (a) Find $\det B$, $\det \frac{1}{3}B$ and $\det B^{-1}$.

   (b) Suppose A is a matrix whose inverse is B. Find the cofactor matrix of A.

8. Find $\begin{vmatrix} 1 & 0 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & 1 \\ 2 & 5 & 0 & 0 & 0 \\ 0 & 0 & 1 & -4 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{vmatrix}$ by reducing to upper triangular form.

9. Let $A = \begin{bmatrix} a & b & c \\ p & q & r \\ u & v & w \end{bmatrix}$ and suppose $\det A = 5$. Find $\begin{vmatrix} 2p & -a+u & 3u \\ 2q & -b+v & 3v \\ 2r & -c+w & 3w \end{vmatrix}$.

10. Let $A = \begin{bmatrix} 5 & -7 & 7 \\ 4 & -3 & 4 \\ 4 & -1 & 2 \end{bmatrix}$; $\mathsf{v}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\mathsf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\mathsf{v}_3 = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$, $\mathsf{v}_4 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $\mathsf{v}_5 = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$ and $\mathsf{v}_6 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$. Determine whether or not the given vectors v_i are eigenvectors of A. Justify your answers.

11. Find the characteristic polynomial and the (real) eigenvalues and corresponding eigenspaces of each of the following matrices.

    (a) $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$

    (b) $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 6 & -6 \\ 1 & 2 & -1 \end{bmatrix}$

12. If x is an eigenvector of an invertible matrix A corresponding to λ, show that x is also an eigenvector of A^{-1}. What is the corresponding eigenvalue?

