Assignment 6 - ANSWERS

120. (a) $\begin{bmatrix} 1 & -1 & -2 \\ 2 & -3 & -5 \\ -1 & 4 & 5 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & -1 & -2 \\ 0 & -1 & -1 \\ 0 & 3 & 3 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & -1 & -2 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{bmatrix}.$

The pivots are 1 and -1. The pivot columns are columns one and two.

(b) $\begin{bmatrix} 1 & 4 & 5 & 2 \\ 3 & 13 & 20 & 8 \\ -2 & -10 & -16 & -4 \\ 1 & 10 & 38 & 28 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 4 & 5 & 2 \\ 0 & 1 & 5 & 2 \\ 0 & -2 & -6 & 0 \\ 0 & 6 & 33 & 26 \end{bmatrix}$

$\longrightarrow \begin{bmatrix} 1 & 4 & 5 & 2 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & 3 & 14 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 4 & 5 & 2 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & 0 & 11 \end{bmatrix}.$

The pivots are 1, 1, 4, and 11. Every column is a pivot column.

121. $\begin{bmatrix} 3 & -1 & 5 & 3 & 6 & -3 \\ 7 & -7 & 1 & 5 & 4 & 2 \\ -4 & 6 & 4 & -2 & 4 & 10 \\ 16 & -10 & 16 & 14 & 22 & -10 \\ -13 & 9 & -11 & -11 & -16 & 12 \end{bmatrix} \longrightarrow \begin{bmatrix} -4 & 6 & 4 & -2 & 4 & 10 \\ 7 & -7 & 1 & 5 & 4 & 2 \\ 3 & -1 & 5 & 3 & 6 & -3 \\ 16 & -10 & 16 & 14 & 22 & -10 \\ -13 & 9 & -11 & -11 & -16 & 12 \end{bmatrix}$

$\longrightarrow \begin{bmatrix} 2 & -3 & -2 & 1 & -2 & -5 \\ 7 & -7 & 1 & 5 & 4 & 2 \\ 3 & -1 & 5 & 3 & 6 & -3 \\ 8 & -5 & 8 & 7 & 11 & -5 \\ -13 & 9 & -11 & -11 & -16 & 12 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & -3 & -2 & 1 & -2 & -5 \\ 0 & \frac{7}{2} & 8 & \frac{3}{2} & 11 & \frac{39}{2} \\ 0 & \frac{7}{2} & 8 & \frac{3}{2} & 9 & \frac{9}{2} \\ 0 & 7 & 16 & 3 & 19 & 15 \\ 0 & -\frac{21}{2} & -24 & -\frac{9}{2} & -29 & -\frac{41}{2} \end{bmatrix}$

$\longrightarrow \begin{bmatrix} 2 & -3 & -2 & 1 & -2 & -5 \\ 0 & 7 & 16 & 3 & 22 & 39 \\ 0 & 7 & 16 & 3 & 18 & 9 \\ 0 & 7 & 16 & 3 & 19 & 15 \\ 0 & -21 & -48 & -9 & -58 & -41 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & -3 & -2 & 1 & -2 & -5 \\ 0 & 7 & 16 & 3 & 22 & 39 \\ 0 & 0 & 0 & 0 & -4 & -30 \\ 0 & 0 & 0 & 0 & -3 & -24 \\ 0 & 0 & 0 & 0 & 8 & 76 \end{bmatrix}$

$\longrightarrow \begin{bmatrix} 2 & -3 & -2 & 1 & -2 & -5 \\ 0 & 7 & 16 & 3 & 22 & 39 \\ 0 & 0 & 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 0 & -4 & -30 \\ 0 & 0 & 0 & 0 & 8 & 76 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & -3 & -2 & 1 & -2 & -5 \\ 0 & 7 & 16 & 3 & 22 & 39 \\ 0 & 0 & 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 12 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & -3 & -2 & 1 & -2 & -5 \\ 0 & 7 & 16 & 3 & 22 & 39 \\ 0 & 0 & 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$

The pivot columns are columns one, two, five, and six.

122. $[A|\mathsf{b}] = \left[\begin{array}{ccc|c} 2 & 2 & 2 & 2 \\ 4 & 6 & 6 & 4 \\ 6 & 6 & 10 & 2 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 4 & 6 & 6 & 4 \\ 6 & 6 & 10 & 2 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 4 & -4 \end{array}\right]$

$\rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 \end{array}\right]$. Thus $x_3 = -1$, $x_2 + x_3 = 0$, so $x_2 = -x_3 = 1$ and $x_1 + x_2 + x_3 = 1$, so $x_1 = 1 - x_2 - x_3 = 1$. The solution is $\mathsf{x} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$.

123. The system is $A\mathsf{x} = \mathsf{b}$ with $A = \begin{bmatrix} -2 & 1 & 5 \\ -8 & 7 & 19 \end{bmatrix}$, $\mathsf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, and $\mathsf{b} = \begin{bmatrix} -10 \\ -42 \end{bmatrix}$. Using Gaussian elimination on the augmented matrix we have

$$[A|\mathsf{b}] = \left[\begin{array}{ccc|c} -2 & 1 & 5 & -10 \\ -8 & 7 & 19 & -42 \end{array}\right] \to \left[\begin{array}{ccc|c} -2 & 1 & 5 & -10 \\ 0 & 3 & -1 & -2 \end{array}\right]$$

so $x_3 = t$ is free,

$3x_2 - x_3 = -2$, so $3x_2 = x_3 - 2 = t - 2$ and $x_2 = \frac{1}{3}t - \frac{2}{3}$,
$-2x_1 + x_2 + 5x_3 = -10$, so $-2x_1 = -x_2 - 5x_3 - 10 = -\frac{16}{3}t - \frac{28}{3}$ and $x_1 = \frac{8}{3}t + \frac{14}{3}$.

The solution is $\mathsf{x} = \begin{bmatrix} \frac{8}{3}t + \frac{14}{3} \\ \frac{1}{3}t - \frac{2}{3} \\ t \end{bmatrix} = \begin{bmatrix} \frac{14}{3} \\ -\frac{2}{3} \\ 0 \end{bmatrix} + t\begin{bmatrix} \frac{8}{3} \\ \frac{1}{3} \\ 1 \end{bmatrix}$.

124. (a) $\left[\begin{array}{cccc|c} 1 & -2 & 3 & -1 & 5 \end{array}\right]$. The free variables are x_2, x_3, and x_4, corresponding to the columns that are not pivot columns.

(b) Let $x_2 = t$, $x_3 = s$, and $x_4 = r$. Then $x_1 - 2x_2 + 3x_3 - x_4 = 5$, so $x_1 = 5 + 2x_2 - 3x_3 + x_4 = 5 + 2t - 3s + r$.

The solution is $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 5 + 2t - 3s + r \\ t \\ s \\ r \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix} + r\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$.

125. $x_3 = t$ is free; $x_4 = \frac{1}{3}$, $x_2 + x_3 + 2x_4 = 3$, so $x_2 = \frac{7}{3} - t$; $x_1 + 3x_4 = 2$, so $x_1 = 1$.

Thus $\mathsf{x} = \begin{bmatrix} 1 \\ \frac{7}{3} - t \\ t \\ \frac{1}{3} \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{7}{3} \\ 0 \\ \frac{1}{3} \end{bmatrix} + t\begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}$. There are infinitely many solutions.

126. (a) $[A|\mathsf{b}] = \left[\begin{array}{ccc|c} 2 & -1 & 2 & -4 \\ 3 & 2 & 0 & 1 \\ 1 & 3 & -6 & 5 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 3 & -6 & 5 \\ 0 & -7 & 14 & -14 \\ 0 & -7 & 18 & -14 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 3 & -6 & 5 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 4 & 0 \end{array}\right]$.

So $z = 0$; $y - 2z = 2$, so $y = 2$; $x + 3y - 6z = 5$, so $x = -1$. The solution is $\begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$.

(b) $[A|\mathsf{b}] = \left[\begin{array}{ccc|c} 1 & -1 & 2 & 4 \end{array}\right]$. This is row echelon form. The free variables are $y = t$ and $z = s$, so $x = 4 + y - 2z = 4 + t - 2s$. In vector form the solution is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 + t - 2s \\ t \\ s \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + s\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}.$$

(c) $[A|\mathsf{b}] = \left[\begin{array}{ccc|c} 2 & -1 & 1 & 2 \\ 3 & 1 & -6 & -9 \\ -1 & 2 & -5 & -4 \end{array}\right] \to \left[\begin{array}{ccc|c} -1 & 2 & -5 & -4 \\ 2 & -1 & 1 & 2 \\ 3 & 1 & -6 & -9 \end{array}\right]$

$$\left[\begin{array}{rrr|r} -1 & 2 & -5 & -4 \\ 0 & 3 & -9 & -6 \\ 0 & 7 & -21 & -21 \end{array}\right] \to \left[\begin{array}{rrr|r} -1 & 2 & -5 & -4 \\ 0 & 1 & -3 & -2 \\ 0 & 1 & -3 & -3 \end{array}\right] \to \left[\begin{array}{rrr|r} -1 & 2 & -5 & -4 \\ 0 & 1 & -3 & -2 \\ 0 & 0 & 0 & -1 \end{array}\right].$$

The last equation reads $0 = -1$. There is no solution.

(d) $[A|\mathsf{b}] = \left[\begin{array}{rrrr|r} -6 & 8 & -5 & -5 & -11 \\ -6 & 7 & -10 & -8 & -9 \\ -8 & 10 & -10 & -9 & -13 \end{array}\right] \to \left[\begin{array}{rrrr|r} 0 & 1 & 5 & 3 & -2 \\ -6 & 7 & -10 & -8 & -9 \\ -8 & 10 & -10 & -9 & -13 \end{array}\right]$

$$\to \left[\begin{array}{rrrr|r} 0 & 1 & 5 & 3 & -2 \\ 2 & -3 & 0 & 1 & 4 \\ -8 & 10 & -10 & -9 & -13 \end{array}\right] \to \left[\begin{array}{rrrr|r} 2 & -3 & 0 & 1 & 4 \\ 0 & 1 & 5 & 3 & -2 \\ -8 & 10 & -10 & -9 & -13 \end{array}\right]$$

$$\to \left[\begin{array}{rrrr|r} 1 & -\frac{3}{2} & 0 & \frac{1}{2} & 2 \\ 0 & 1 & 5 & 3 & -2 \\ -8 & 10 & -10 & -9 & -13 \end{array}\right] \to \left[\begin{array}{rrrr|r} 1 & -\frac{3}{2} & 0 & \frac{1}{2} & 2 \\ 0 & 1 & 5 & 3 & -2 \\ 0 & -2 & -10 & -5 & 3 \end{array}\right]$$

$$\to \left[\begin{array}{rrrr|r} 1 & -\frac{3}{2} & 0 & \frac{1}{2} & 2 \\ 0 & 1 & 5 & 3 & -2 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right].$$

The variable $x_3 = t$ is free. Back substitution gives

$x_4 = -1$,

$x_2 + 5x_3 + 3x_4 = -2$, so

$x_2 = -2 - 5x_3 - 3x_4 = -2 - 5t + 3 = 1 - 5t$, and then

$x_1 - \frac{3}{2}x_2 + \frac{1}{2}x_4 = 2$, so

$$\begin{aligned} x_1 &= 2 + \tfrac{3}{2}x_2 - \tfrac{1}{2}x_4 = 2 + \tfrac{3}{2}(1-5t) - \tfrac{1}{2}(-1) \\ &= 2 + \tfrac{3}{2} - \tfrac{15}{2}t + \tfrac{1}{2} = 4 - \tfrac{15}{2}t. \end{aligned}$$

The solution is $\begin{bmatrix} 4 - \frac{15}{2}t \\ 1 - 5t \\ t \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 0 \\ -1 \end{bmatrix} + t \begin{bmatrix} -\frac{15}{2} \\ -5 \\ 1 \\ 0 \end{bmatrix}.$

127. We attempt to solve $A\mathsf{x} = \mathsf{b}$, with $\mathsf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $\mathsf{b} = \begin{bmatrix} 1 \\ 6 \\ -4 \end{bmatrix}$. Gaussian elimination proceeds

$$[A|\mathsf{b}] = \left[\begin{array}{rrr|r} 2 & 3 & 4 & 1 \\ 4 & 7 & 5 & 6 \\ 6 & -1 & 9 & -4 \end{array}\right] \to \left[\begin{array}{rrr|r} 2 & 3 & 4 & 1 \\ 0 & 1 & -3 & 4 \\ 0 & -10 & -3 & -7 \end{array}\right] \to \left[\begin{array}{rrr|r} 2 & 3 & 4 & 1 \\ 0 & 1 & -3 & 4 \\ 0 & 0 & -33 & 33 \end{array}\right]$$

$\to \left[\begin{array}{rrr|r} 2 & 3 & 4 & 1 \\ 0 & 1 & -3 & 4 \\ 0 & 0 & 1 & -1 \end{array}\right]$, so $x_3 = -1$, $x_2 = 4 + 3x_3 = 1$, $2x_1 = 1 - 3x_2 - 4x_3 = 2$, and

$x_1 = 1$. Thus $A\begin{bmatrix}1\\1\\-1\end{bmatrix} = \begin{bmatrix}1\\6\\-4\end{bmatrix}$. This says that $\begin{bmatrix}1\\6\\-4\end{bmatrix} = 1\begin{bmatrix}2\\4\\6\end{bmatrix} + 1\begin{bmatrix}3\\7\\-1\end{bmatrix} - \begin{bmatrix}4\\5\\9\end{bmatrix}$.

128. The question asks if there are scalars a and b such that $\begin{bmatrix}2\\-11\\-3\end{bmatrix} = a\begin{bmatrix}0\\-1\\5\end{bmatrix} + b\begin{bmatrix}-1\\4\\9\end{bmatrix}$. This is $\begin{bmatrix}0&-1\\-1&4\\5&9\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}2\\-11\\-3\end{bmatrix}$. Gaussian elimination proceeds

$$\left[\begin{array}{rr|r}0&-1&2\\-1&4&-11\\5&9&-3\end{array}\right] \to \left[\begin{array}{rr|r}1&-4&11\\0&-1&2\\0&29&-58\end{array}\right] \to \left[\begin{array}{rr|r}1&-4&11\\0&1&-2\\0&0&0\end{array}\right].$$

There is a unique solution: $b = -2$, $a = 3$. The given vector is indeed a linear combination of the other two:. $\begin{bmatrix}2\\-11\\-3\end{bmatrix} = 3\begin{bmatrix}0\\-1\\5\end{bmatrix} - 2\begin{bmatrix}-1\\4\\9\end{bmatrix}$.

129. The question asks whether every $\mathsf{b} = \begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix}$ in R^3 is a linear combination of the given vectors. The vector equation $c_1\mathsf{v}_1+c_2\mathsf{v}_2+c_3\mathsf{v}_3+c_4\mathsf{v}_4 = \mathsf{b}$ gives the system of equations

$$\begin{aligned} 2c_1 + c_2 - c_3 \phantom{{}+2c_4} &= b_1\\ c_2 + c_3 + 2c_4 &= b_2\\ 2c_1 + c_2 - c_3 \phantom{{}+2c_4} &= b_3. \end{aligned}$$

Gaussian elimination proceeds

$$\left[\begin{array}{rrrr|l}2&1&-1&0&b_1\\0&1&1&2&b_2\\2&1&-1&0&b_3\end{array}\right] \to \left[\begin{array}{rrrr|c}1&\frac12&-\frac12&0&\frac12 b_1\\0&1&1&2&b_2\\0&0&0&0&b_3-b_1\end{array}\right].$$

This system has a solution if and only if $b_3 - b_1 = 0$. So not every vector in R^3 is a linear combination of the given vectors since not all vectors in R^3 satisfy this condition. In fact, the given vectors span the plane with equation $x - z = 0$.

130. The question is whether or not these points all satisfy an equation of the form $ax + by + cz + d = 0$. Substituting the coordinates of each point in turn, we get the system

$$\begin{aligned} -2a + b + c + d &= 0\\ 2a + 2b - 2c + d &= 0\\ 4a + 7b - c + d &= 0\\ -10a + 11b - 9c + d &= 0. \end{aligned}$$

Gaussian elimination proceeds

$$\begin{bmatrix} -2 & 1 & 1 & 1 \\ 2 & 2 & -2 & 1 \\ 4 & 7 & -1 & 1 \\ -10 & 11 & -9 & 1 \end{bmatrix} \to \begin{bmatrix} -2 & 1 & 1 & 1 \\ 0 & 3 & -1 & 2 \\ 0 & 9 & 1 & 3 \\ 0 & 6 & -14 & -4 \end{bmatrix}$$

$$\to \begin{bmatrix} -2 & 1 & 1 & 1 \\ 0 & 3 & -1 & 2 \\ 0 & 0 & 4 & -3 \\ 0 & 0 & -12 & -6 \end{bmatrix} \to \begin{bmatrix} -2 & 1 & 1 & 1 \\ 0 & 3 & -1 & 2 \\ 0 & 0 & 4 & -3 \\ 0 & 0 & 0 & -15 \end{bmatrix}$$

The equation corresponding to the last row of the final matrix reads $0 = -15$. There is no solution. The points are not coplanar.

131. (a) The normals to the planes, $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$, are not parallel, so the planes are not parallel.

(b) We solve the system $\begin{aligned} x \quad + 2z &= 5 \\ 2x + y \quad &= 2. \end{aligned}$

By Gaussian elimination, $\left[\begin{array}{ccc|c} 1 & 0 & 2 & 5 \\ 2 & 1 & 0 & 2 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 2 & 5 \\ 0 & 1 & -4 & -8 \end{array}\right]$,

so $z = t$ is free, $y - 4z = -8$, so $y = 4t - 8$ and $x + 2z = 5$, so $x = -2t + 5$. The solution is $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2t+5 \\ 4t-8 \\ t \end{bmatrix} = \begin{bmatrix} 5 \\ -8 \\ 0 \end{bmatrix} + t\begin{bmatrix} -2 \\ 4 \\ 1 \end{bmatrix}$. This is the equation of the line of intersection.

132. (a)

$$\begin{aligned} 3 &= a - 2b + 4c \\ -11 &= a \\ 24 &= a + 5b + 25c \end{aligned}$$

(b) $a = -11$, $b = -3$, $c = 2$; thus $p(x) = -11 - 3x + 2x^2$.

133. Gaussian elimination begins $\left[\begin{array}{cc|c} 5 & 2 & a \\ -15 & -6 & b \end{array}\right] \to \left[\begin{array}{cc|c} 5 & 2 & a \\ 0 & 0 & b+3a \end{array}\right]$.

(a) If $b + 3a \neq 0$, the second equation is $0 = b + 3a \neq 0$, so there is no solution.

(b) Under no circumstances does the system have a unique solution since if $b + 3a = 0$, y is free and there are infinitely many solutions.

(c) If $b + 3a = 0$, there are infinitely many solutions.

Row echelon form is $\left[\begin{array}{cc|c} 1 & \frac{2}{5} & \frac{a}{5} \\ 0 & 0 & 0 \end{array}\right]$, so $y = t$ is free and $x = \frac{a}{5} - \frac{2}{5}y = \frac{a}{5} - \frac{2}{5}t$.

134. (a) $\left[\begin{array}{rr|c} 1 & -2 & a \\ -5 & 3 & b \\ 3 & 1 & c \end{array}\right] \to \left[\begin{array}{rr|c} 1 & -2 & a \\ 0 & -7 & b+5a \\ 0 & 7 & c-3a \end{array}\right] \to \left[\begin{array}{rr|c} 1 & -2 & a \\ 0 & -7 & b+5a \\ 0 & 0 & (c-3a)+(b+5a) \end{array}\right]$

The system has a solution if and only if $(c-3a)+(b+5a)=0$; that is, if and only if $2a+b+c=0$.

(b) If $2a+b+c=0$, row echelon form is $\begin{bmatrix} 1 & -2 & a \\ 0 & -7 & b+5a \\ 0 & 0 & 0 \end{bmatrix}$, which implies a unique solution: $y=-\frac{1}{7}(b+5a)$, and so on. Infinitely many solutions is not a possibility.

135. This system is homogeneous. The elementary row operations will not change the right column of 0s, so we apply Gaussian elimination to the matrix of coefficients (not the **augmented** matrix of coefficients), simply remembering that there is a final column of 0s.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 0 \\ 2 & -1 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & -1 \\ 0 & -3 & -3 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & -3 \\ 0 & 2 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$
$$\to \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -3 \end{bmatrix}.$$

Remembering that the constants are all 0, the last equation reads $-3x_3=0$, so $x_3=0$, then $x_2+x_3=0$ gives $x_2=0$ too and $x_1+x_2+x_3=0$ gives $x_1=0$. The solution is the zero vector, $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

136. (a) We seek a,b,c, not all 0, such that

$$a\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + b\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + c\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \qquad (*)$$

This leads to the system of equations

$$\begin{aligned} a-b+\ c &= 0 \\ b+\ c &= 0 \\ a\quad +2c &= 0. \end{aligned}$$

The solutions are $a=-2t$, $b=-t$, $c=t$. Thus, and for example, $a=-2$, $b=-1$, $c=1$ is a solution to (*).

(b) The result of (a) shows that $A\mathsf{x}=0$, where $\mathsf{x}=\begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$. If A were invertible, we would have $A^{-1}A\mathsf{x}=A^{-1}0$; that is, $\mathsf{x}=0$, which is not true. Thus A is not invertible.