Assignment1 Solutions

Problem 1: Classify each of the following statements as true or false and explain your answers.

(b.)"$4 \neq 2+2 \quad and \quad 7 < \sqrt{50}$"

(d.)"$4 = 2+2 \leftrightarrow 7 < \sqrt{50}$"

(f.)"$4 \neq 2+2 \leftrightarrow 7 < \sqrt{50}$"

(h.) "The area of a circle of radius r is $2\pi r$ or its circumference is πr^2"

Solution:

(b.) This statement is **False**.

Set "$4 \neq 2+2$" to p, and "$7 < \sqrt{50}$" to q. So p is False and q is True.

Therefore, $p \wedge q$ is False.

(d.) This statement is **True.**

Since "$4 = 2+2$" is True and "$7 < \sqrt{50}$" is True.

Therefore, "$4 = 2+2 \leftrightarrow 7 < \sqrt{50}$" is True.

(f.) This statement is **False**.

Since "$4 \neq 2+2$" is False and "$7 < \sqrt{50}$" is True.

Therefore, "$4 \neq 2+2 \leftrightarrow 7 < \sqrt{50}$" is False.

The only way that the biconditional $q \leftrightarrow p$ is true is when both statement have the same truth value.

(h.)This statement is **False**.

Since the statement "The area of a circle of radius r is $2\pi r$ is False and "its circumference is πr^2" is False.

This therefore causes $p \vee q$ to be False.

Problem 2: Clasify each of the following statements as true or false and explain your answers.

(b.) If a and b are integers with $a - b > 0$ and $b - a > 0$, then $a = b$.

Solution:
This statement is **True**.
Since a and b are both integers then both $a - b > 0$ and $b - a > 0$ are false.
Therefore, $p \to q$ is true, since p is false it does not matter whether q is true or false.

Problem 3: Write down the negation of each of the following statements in clear and concise English.
(b.) x is a real number and $x^2 + 1 = 0$.
(d.) Every integer is divisible by a prime.
(f.) There exists a, b, and c such that $(ab)c \neq a(bc)$.

Solution:
(b.) x is not a real number or $x^2 + 1 \neq 0$ (Use DeMorgan's Law)
(d.) There is an integer which is not divisble by a prime
(f.) For all a, b, and c, $(ab)c = a(bc)$. This was determined using $\neg(\exists x p(x)) \leftrightarrow \forall x \neg p(x)$

Problem 4: Write down the converse and the contrapositive of each of the following implications.
(d.) $ab = 0 \to a = 0$ or $b = 0$.
(f.) If $\triangle BAC$ is a right triangle, then $a^2 = b^2 + c^2$.

Solution:
(d.)**Converse:** $a = 0$ or $b = 0 \to ab = 0$
Contrapositive: $a \neq 0$ and $b \neq 0 \to ab \neq 0$ (DeMorgan's Law)

(f.) **Converse:** If $a^2 = b^2 + c^2$, then $\triangle BAC$ is a right triangle.
Contrapositive: If $a^2 \neq b^2 + c^2$, then $\triangle BAC$ is not a right triangle.

Problem 5: Rewrite each of the following statements using the quantifiers "for all" and "there exists" as appropriate.
(b.) For real x, 2^x is never negative.
(f.) All positive real numbers have real square roots.

Solution:
(b.) There is no real x such that 2^x is negative.
(f.) For all positive real numbers its square root is real.

Problem 6: Construct a truth table for each of the following compound statements.
(c.) $\neg(p \wedge (q \vee p)) \leftrightarrow p$.
(e.) $(p \to (q \to r)) \to ((p \wedge q) \vee r$.

Solution:

(c.)

p	q	$q \vee q$	$p \wedge (q \vee p)$	$\neg(p \wedge (q \vee p))$	$\neg(p \wedge (q \vee p)) \leftrightarrow p$
T	T	T	T	F	F
T	F	T	T	F	F
F	T	T	F	T	F
F	F	F	F	T	F

(e.)

p	q	r	$q \to r$	$p \to (q \to r)$	$p \wedge q$	$(p \wedge q) \vee r$	$(p \to (p \to r)) \to ((p \wedge q) \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	T	T
T	F	T	T	T	F	T	T
T	F	F	T	T	F	F	F
F	T	T	T	T	F	T	T
F	T	F	F	T	F	F	F
F	F	T	T	T	F	T	T
F	F	F	T	T	F	F	F

Problem 7: Determine the truth value for $[p \to (q \wedge (\neg r))] \vee [r \leftrightarrow ((\neg s) \vee q)]$ in the case where p, q, r and s are all false.

Solution:

p	q	r	s	$\neg r$	$q \wedge (\neg r)$	$p \to (q \wedge (\neg r)$	$\neg s$	$\neg s \vee q$	$r \leftrightarrow ((\neg s) \vee q)$
F	F	F	F	T	F	T	T	T	F

Therefore $[p \to (q \wedge (\neg r))] \vee [r \leftrightarrow ((\neg s) \vee q)]$ is **True**.

Problem 8:
(a.) Show that $q \to (p \to q)$ ia a tautology.
(b.) Show that $[p \wedge q] \wedge [(\neg p) \vee (\neg q)]$ is a contradiction.

Solution:

(a.)

p	q	$p \to q$	$q \to (p \to q)$
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	T

$q \to (p \to q)$ is always True, which means we have a tautology.

(b.)

p	q	$p \wedge q$	$\neg p$	$\neg q$	$(\neg p) \vee (\neg q)$	$[p \wedge q] \vee [(\neg p) \vee (\neg q)]$
T	T	T	F	F	F	F
T	F	F	F	T	T	F
F	T	F	T	F	T	F
F	F	F	T	T	T	F

$[p \wedge q] \wedge [(\neg p) \vee (\neg q)]$ is always False, which means we have a contradiction.

Problem 9: If p and q are statements, then the compound statement $p \veebar q$ (often called the exclusive or) is defined to be true if and only if exactly one of p, q is true: that is either p is true or q is true, but not both p and q are true.

(d.) Show that $p \veebar q$ is logically equivalent to $\neg(p \leftrightarrow q)$

Solution:

p	q	$p \veebar q$	$p \leftrightarrow q$	$\neg(p \leftrightarrow q)$
T	T	F	T	F
T	F	T	F	T
F	T	T	F	T
F	F	F	T	F

$p \veebar q$ and $\neg(p \leftrightarrow q)$ have the same truth table. So therefore they are logically equivalent, that is $p \veebar q \longleftrightarrow \neg(p \leftrightarrow q)$.