MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SAMPLE FINAL EXAM MATHEMATICS 2320 APRIL, 2005

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1. Find converse, contrapositive and negation of statement:

If it snows tonight, I will stay at home.

2. Give an example of an open statement and its existential quantification.

3. Construct thruth table for $(x \wedge y) \vee (\neg z \to x)$. Is it tautology or contradiction or none?

4. Is the following logical argument valid?

If you do every problem from this sample test then you will learn discrete math.

You learned discrete math.

Therefore you did every problem from this test.

5. Prove that $a^2 = b^2$ if and only if $a = b$ or $a = -b$.

6. Prove that n is even if and only if $7n + 4$ is even.

7. Prove that $1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \cdots + n \cdot n! = (n+1)! - 1$ for all positive integer n.

8. Find the power set $\mathcal{P}(S)$ of set $S = \{\, a, \{a\} \,\}$. What is cardinality of the Cartesian product $S \times \mathcal{P}(S)$?

9. Show that that the set of rational numbers is countable.

10. Give an example of a finite non-empty partially ordered set without the maximum element.

11. Let a binary relation be defined as:

two people are in the relation if their first names have the same number of letters.

Is this an equivalence relation or a partial order? If this is an equivalence relation, then what is the class of equivalence of *John*?

12. Is the function injective? surjective? bijective?

a) $f : Z \to Z_+$, $f(x) = x^4$. (here Z is the set of integers, Z_+ - non-negative integers).

b) $f : R \to R$, $f(x) = 2^x$. (here R is the set of real nambers).

c) $f : N \to N$, $f(x) = x + 5$ (here N is the set of natural nambers).

13. Find sets A and B such that function $f : A \to B$,$f = (x-1)^{-1/2}$ is a bijection. Find its inverse.

14. A bowl contains 10 red balls and 10 blue balls. A women selects balls at random without looking at them.

a) how many balls must she select to be sure of having at least three balls of the same color?

b) how many balls must she select to be sure of having at least three blue balls?

15. English alphabet contains 21 consonants and five vowels. How many strings of six lowercase letters of the English alphabet contain

a) exactly 1 vowel ?

b) at least one vowel ?

16. What is the coefficient of x^9 in $(2-x)^{19}$?

17. How many ways are there to distribute hands of 5 cards to each of four players from the standard deck of 52 cards?

18. Solve $a_{n+2} = 6a_{n+1} - 9a_n$, $a_0 = 1$, $a_1 = 2$.