MUN Undergraduate Mathematics Competition. April 2010.

This Take Home Competition consists of 7 problems and is open to all undergraduate students. Authors of the best papers will be considered for participation in the Fall 2010 APICS mathematics competition and conference.

Submit your solution to ***Margo Kondratieva (HH-3008 or mkondra@mun.ca)***

Your papers are **due April 9, 2010.**

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1. Simplify

$$\frac{\tan 1}{\cos 2} + \frac{\tan 2}{\cos 4} + \frac{\tan 4}{\cos 8} + \cdots + \frac{\tan 128}{\cos 256}$$

2. Let G be a group generated by a and b subject to the relation $aba = b^3$ and $b^5 = 1$. Prove that G is abelian.

3. Four distinct points in the plane have the property that any three of them can be covered by a circle of radius one. Prove that all four of them can be covered by a circle of radius one.

4. a) Show that if a number is a multiple of 6 then it can be written as a sum of cubes of four integers. For example, $6 = (-1)^3 + (-1)^3 + 0^3 + 2^3$.

b) Show that any integer number can be written as a sum of cubes of five integers.

5. a) Show that

$$\lim_{x\to 0} \frac{(1+x)^{1/x} - e}{x} = -\frac{e}{2}$$

b) Find the limit

$$\lim_{x\to 0} \frac{(1+x)^{1/x} - e + ex/2}{x^2}$$

6. Let $A = ((a_{ij}))$ be real symmetric (2×2) - matrix with eigenvalues λ_1 and λ_2. Find the maximum and minimum of possible values of the matrix element a_{12}.

7. Let ABCD be a parallelogram with AB=CD being the longest sides. Let G be a midpoint of side CD and point H be the point of intersection of the segment AG (or its extension) and the perpendicular line dropped from point B on line AG. Prove that BC=CH.