## MUN Undergraduate Mathematics Competition. April 2009.

This Take Home Competition consists of 7 problems and is open to all undergraduate students. Authors of the best papers will be considered for participation in the Fall 2009 APICS mathematics competition and conference.

Submit your solution to Margo Kondratieva (MUN HH-3008 or mkondra@mun.ca) Your papers are due April 8, 2009.

1. Find all solutions of the equation

$$
M A T H=A M A T+P M A T
$$

where different letters represent different digits.
2. (a) Prove that the equation

$$
x^{4}+y^{4}+z^{4}-2 y^{2} z^{2}-2 z^{2} x^{2}-2 x^{2} y^{2}=24
$$

has no integer solutions.
(b) Does this equation have rational solutions? If yes, give an example. If no, prove it.
3. Take a rectangular piece of paper. Place it on the table in front of you. (1) Fold it across a main diagonal. (2) Unfold it, and fold it in half parallel to the shorter side.

(3) Unfold again and fold across the main diagonal of one of the smaller rectangles. The pair of diagonals crosses at a point $P$. Prove that a line through this point, parallel to either side of the original paper, divides the paper exactly into thirds (shortways or lengthways).
4. Evaluate the infinite sum

$$
S=1-\frac{2^{3}}{1!}+\frac{3^{3}}{2!}-\frac{4^{3}}{3!}+\ldots
$$

5. Prove that for any $x>0$ and $y>0$

$$
\lim _{p \rightarrow \infty}\left(\frac{x^{1 / p}+y^{1 / p}}{2}\right)^{p}=\sqrt{x y}
$$

6. Assume $f(x)=\frac{1}{1-x-x^{2}}$ and $a_{n}=\frac{1}{n!} f^{(n)}(0)$. Prove that the following series is convergent and find the sum:

$$
\sum_{n=0}^{\infty} \frac{a_{n+1}}{a_{n} a_{n+2}}
$$

7. (a) During the year 2008 a convenience store, which was open every day of the year, sold at least one book a day, and a total of 601 books over the entire year. Prove that there was a period of consecutive days when exactly 129 books were sold.
(b) Would the result in (a) still be true if the year were 2009 instead of 2008? Either give a proof or provide a counterexample.
