Math 3210 **Due Nov 17** **Assignment #8**

1. Explain how the fundamental theorem of algebra follows from the Liouville's theorem.
2. Explain how Maximum Modulus Principle follows from Gauss' Mean Value theorem.
3. Evaluate the contour integral.

   (a) $\oint_{|z|=3} \frac{cosh(z)}{(z-i)^4}\, dz.$

   (b) $\oint_{C} \frac{e^{-2z}}{(z-1)^3}\, dz$ where C is a rectangle with vertices at $\pm 2 \pm 2i$.

   (c) $\oint_{|z|=1} \frac{\tan(z/2)}{(z+\pi/2)^2}\, dz.$

   (d) $\oint_{|z|=2} \frac{\tan(z/2)}{(z+\pi/2)^2}\, dz.$

4. Let $0 < a < R < b$ be real numbers. Evaluate

$$\oint_{|z|=R} \frac{1}{(z-a)^n(z-b)}\, dz, \quad n = 1, 2, 3...$$

5. Let

$$F_n(z) = \frac{1}{n!2^n}\frac{d^n}{dz^n}(z^2-1)^n, \quad n = 0, 1, 2, 3, ...$$

   (a) show that $F_n(z)$ is a polynomial of degree n. (These are Legendre polynomials.)

   (b) show that $F_n(z)$ can be expressed in the integral form

$$F_n(z_0) = \frac{1}{2^{n+1}\pi i} \oint_{|z-z_0|=r} \frac{(z^2-1)^n}{(z-z_0)^{n+1}}\, dz.$$

   (c) use (b) to find $F_n(\pm 1)$.

6. Consider $f(z) = e^z$ and rectangular region R: $0 \le x \le 1$, $0 \le y \le \pi$.

   a) Find maximum and minimum values of $u(x, y) = Re(f(z))$ in the region R as well as the point where each extremum is reached.

   b) Find maximum and minimum values of $v(x, y) = Im(f(z))$ in the region R as well as the point where each extremum is reached.

7. Let $f(z) = u(x, y) + iv(x, y)$ be continuous on a closed bounded region R. Suppose it is analytic but not constant in the interior of R. Show that functions $u(x, y)$ and $v(x, y)$ both have their max and minimum values on the boundary of R but not in the interior.