Math 3210 **Due Thur Oct 15** **Assignment #4**

1. Show that function $v(x, y)$ is harmonic and find the function $u(x, y)$ of which it is harmonic conjugate.

$$(a) v = 3xy + 5x \qquad (b) v = y^4 - 6x^2y^2 + x^4 - y \qquad (c) v = e^{-2x}\cos(2y)$$

2. Let $F(z)$ be analytic in D. Prove that if $F(z)$ is real valued in D then $F(z)$ must be a constant in D.

3. **Lemma** Let $f(z) = u(x, y) + iv(x, y)$ be analytic. Let curves $u(x, y) = c_1$ and $v(x, y) = c_2$ intersect at point z_0 and $f'(z_0) \neq 0$. Then the lines tangent to the curves at z_0 are perpendicular.

   Illustrate the Lemma for $f(z) = \frac{z-1}{z+1}$, $z \neq -1$ by sketching few curves $u(x, y) = c_1$ and $v(x, y) = c_2$. Also, explain why this function is analytic.

4. Prove the following theorem.

   Theorem Let $f(z)$ be analytic in the complex plane. Then $\overline{f(z)} = -f(\bar{z})$ if and only if $f(x)$ is pure imaginary.

5. Check if the following function satisfy $\overline{f(z)} = \pm f(\bar{z})$ by two ways: directly and using reflection principles.

$$a) f(z) = z^8 \qquad b) f(z) = iz^7 \qquad c) f(z) = cosz \qquad d) f(z) = cos(iz)$$

6. Evaluate (show all steps)

$$a) \exp(\tfrac{3+i\pi}{6}), \qquad b) (\exp(z^5))' \text{ at } z = 2i, \qquad c) \log(i^2)$$

$$d) \log(i^{1/2}) \qquad e) \log e, \qquad f) \log(1 + \sqrt{3}i).$$

7. a) Find ALL values of z such that e^z is pure imaginary.

   a) Find ALL values of z such that $e^{i\bar{z}} = \overline{e^{iz}}$

8. Solve for z (show all steps)

   (a) $\log(2z) = i\pi/2$
   (b) $\tan(z/2) = 2i$
   (c) $\sinh z = 2i$
   (d) $\sinh(\cos z) = 0$

9. **Extra Points Problem** Let $f(z) = u(r, \theta) + iv(r, \theta)$ be analytic. Find 2nd order partial differential equation for function $u(r, \theta)$. How does it relate to the Laplace equation we considered before?