Math 3210 **Due Nov 23** **Assignment #9**

1. Write down the most important statements you know about series representation for analitic functions. Type your favorite formula from the course in the Discussion (Message) Board at my web page.

   http://www.math.mun.ca/ mkondra/okno/index.php Please, select Math 3210

2. Let $\sum_{n=0}^{\infty} z_n = S$ and $\sum_{n=0}^{\infty} w_n = T$. Find $\sum_{n=0}^{\infty} \bar{z}_n$, $\sum_{n=0}^{\infty} a w_n$, $\sum_{n=0}^{\infty} (i z_n + 3\bar{w}_n)$.

3. True of false?
$$\sum_{n=1}^{\infty} r^n \sin(nt) = \frac{r \sin t}{1 - 2r\cos t + r^2}.$$

4. Find series expantion in terms of integer powers of $z-a$ for given function $f(z)$ and complex number a. Find domain of convergence. Which of them are power series?

   (a) $f(z) = e^z$, $a = \pi i$.
   (b) $f(z) = z^6 \sinh(z^{-3})$, $a = 0$.
   (c) $f(z) = \sin(z - \pi/2)$, $a = 0$.
   (d) $f(z) = \cosh z$, $a = i\pi/2$.
   (e) $f(z) = (5z - z^2)^{-1}$, $a = 0$.

5. Find Laplace series representation centered at zero in each domain of analitycity for the function
$$f(z) = \frac{2z-3}{(z-5)(z+2)}.$$

6. Show that function $f(z) = (1+z^2)^{-1}$, $z \neq \pm i$ is an analytic continuation of $g(z) = \sum_{n=0}^{\infty} (-1)^n z^{2n}$ from the disk $|z| < 1$ to the complex plane without points $\pm i$.

7. Consider function $f(z) = \dfrac{\cos z}{z^2 - (\pi/2)^2}$. This function has removable discontinuity at points $z = \pm\pi/2$. What values should be assigned at the poins to make the function continuous? Does the continuous function become entire? Explain.

8. Let z be a complex number. Show that
$$\exp\left(\frac{z}{2}\left(w - \frac{1}{w}\right)\right) = \sum_{n=-\infty}^{\infty} J_n(z) w^n, \quad 0 < |w| < \infty$$
   where $J_n(z)$ are Bessel functions of the first kind, namely
$$J_n(z) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-i(nt - z\sin t)}\, dt, \quad n = 0, \pm 1, \pm 2, ...$$
   Hint: Use integral formula for coefficients in Laurent series and unit contour of integration $w = e^{it}$.

   Extra point problem. Show that Bessel function $J_n(z)$ solves the Bessel equation
$$z^2 y'' + z y' + (z^2 - n^2) y = 0.$$
   Show that Bessel equation appears when you solve the following Helmholz equation
$$U_{xx} + U_{yy} = -k^2 U, \quad k = \text{const}$$
   by the separation of variables method in polar coordinates.