Math 3210 **Due Thur Oct 14** **Assignment #4**

1. Show that function $v(x, y)$ is harmonic and find the function $u(x, y)$ of which it is harmonic conjugate.

   (a) $v = 2y - 3x^2y + y^3$
   (b) $v = \cos y \cosh x$
   (c) $v = x/(x^2 + y^2)$

2. Show that $F(z)$ is analytic in D if and only if $-iF(z)$ is analytic there.

3. **Lemma** Let $f(z) = u(x, y) + iv(x, y)$ be analytic. Let curves $u(x, y) = c_1$ and $v(x, y) = c_2$ intersect at point z_0 and $f'(z_0) \neq 0$. Then the lines tangent to the curves at z_0 are perpendicular.

   (a) Demonstrate the Lemma for $f(z) = z^{-1}$, $z \neq 0$;
   (b) Prove the Lemma. Hint: regard y as a function of x and differtiate equations of the curves w.r.t. x. Then, find product of slopes of tangent lines at the intersection point. Use C.-R. equations to show that the product is -1. Make conclusion.

4. Consider functions
   $f_1(z) = \sqrt{z}e^{\theta/2}$, $r > 0$, $0 < \theta < \pi$;
   $f_2(z) = \sqrt{z}e^{\theta/2}$, $r > 0$, $\pi/2 < \theta < 2\pi$;
   $f_3(z) = \sqrt{z}e^{\theta/2}$, $r > 0$, $3\pi/2 < \theta < 5\pi/2$;
   Show that f_2 is analytic continuation of f_1, f_3 is analytic continuation of f_2, but $f_3 \neq f_1$ on their common domain.

5. Check if the following function satisfy $f(\bar{z}) = f(\bar{z})$ by two ways: directly and using reflection principle.

   $a) f(z) = z^3$ $\qquad b) f(z) = z^3(1 + i)$

   $c) f(z) = e^z$ $\qquad d) f(z) = e^{iz}$

6. Evaluate

   (a) $\exp(\frac{2+i\pi}{4})$
   (b) $(\exp(z^4))'$ at $z = -i$
   (c) $\log i$
   (d) $\log e$
   (e) $\log(1 + \sqrt{3}i)$

7. Solve for z

   (a) $\log z = i\pi/2$
   (b) $\tan z = 2i$
   (c) $\sinh z = i$
   (d) $\sinh(\cos z) = 0$

8. Show that $\text{Log}(1 + i)^2 = 2\text{Log}(1 + i)$, but $\text{Log}(-1 + i)^2 \neq 2\text{Log}(-1 + i)$.

9. **Extra Points Problem** Let $f(z) = u(r, \theta) + iv(r, \theta)$ be analytic. Find 2nd order partial differetial equation for function $u(r, \theta)$. How does it relate to the Laplace equation we considered before?