Math 2000: Assignment #1, Due Jan 20. W-2006

1. Find the first 5 terms of following sequences.

a) $a_n = \dfrac{n+1}{3n+1}$ b) $a_1 = 4, \quad a_{n+1} = \dfrac{a_n}{a_n - 1}$

2. Find a formula for the general term a_n of the following sequences, assuming that the pattern of the first few terms continues.

a) $\left\{\frac{1}{2}, \frac{3}{4}, \frac{9}{8}, \frac{27}{16}, \frac{81}{32}, \frac{243}{64} \ldots\right\}$ b) $\left\{\frac{1}{2}, -\frac{3}{4}, \frac{5}{6}, -\frac{7}{8}, \frac{9}{10}, -\frac{11}{12} \ldots\right\}$

3. Determine if the following sequences converge or diverge. Find the limit of convergent sequences.

a) $a_n = \dfrac{n+1}{3n-1}$ b) $b_n = \dfrac{3^n}{n!}$ c) $c_n = (-1)^n \dfrac{n}{n+1}$

d) $b_n = \dfrac{n!}{(n+1)!}$ e) $c_n = \cos\left(\dfrac{2}{n}\right)$ f) $a_n = \dfrac{\sqrt{n}}{\sqrt{n+1}}$

g) $c_n = \dfrac{(-3)^n}{(n+1)!}$ h) $a_n = \arctan(2n)$ i) $b_n = \dfrac{\ln(x)}{\ln(2x)}$

4. Determine whether the following sequences are increasing, decreasing, or not monotonic. Which ones are bounded?

a) $a_n = \dfrac{1}{3^n}$ b) $b_n = \dfrac{2n-3}{3n+4}$ c) $c_n = \dfrac{1}{n}\sin\left(\dfrac{\pi n}{4}\right)$

5. Show that the sequence defined by

$$a_1 = 2, \quad a_{n+1} = \frac{1}{3 - a_n}$$

satisfies $0 \leq a_n \leq 2$ and is decreasing. Deduce that the sequence is convergent and find its limit.

6. Find the first 5 partial sums of the following series. Is the series convergent or divergent? Explain.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2 - 1}{3n^2 + 1}$ b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{3^{n-1}}$