**Math 1001 Sec. 1 (Margo) TEST #2 Answers Nov 18, 2008**

| **NAME:** | **Student #:** |
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**Problem 1** (25 points)

Let a "triangular" region be bounded by curves given by their equations:

$$y = 2x^2, \quad y = 8, \quad y = -4x.$$

a) Sketch the region and mark all curves and points of their intersections

**Answer:** The region is bounded by three curves:
a line segment connecting point (-2,8) and the origin (0,0);
horizontal line connecting points (-2,8) and (2,8) ;
a segment of parabola connecting points (2,8) and the origin.

b) Find the area of the region by integrating with respect to x-variable

**Answer:**

$$\int_{-2}^{0} 8 - (-4x)\, dx + \int_{0}^{2} 8 - (2x^2)\, dx = 8 + \frac{32}{3} = \frac{56}{3}$$

c) Find the area of the region by integrating with respect to y-variable. Compare your answer with (b).

**Answer:**

$$\int_{0}^{8} \sqrt{\frac{y}{2}} - (-\frac{y}{4})\, dy = \frac{56}{3}$$

d) Find the volume of the solid of revolution obtained by revolving the region about the line $y = 8$. Show your work: set up the integral and evaluate it.

**Answer:** Disk method for two parts separately.

$$\pi \int_{-2}^{0} (8 - (-4x))^2\, dx + \pi \int_{0}^{2} (8 - (2x^2))^2\, dx = \frac{1664\pi}{15}$$

e) Find the volume of the solid of revolution obtained by revolving the region about the line $x = 2$. Show your work: set up the integral and evaluate it.

**Answer:** Washer method.

$$\pi \int_{0}^{8} \left(2 - (-\frac{y}{4})\right)^2 - \left(2 - \sqrt{\frac{y}{2}}\right)^2\, dy = \frac{208\pi}{3}$$

**Problem 2** (35 points) Evaluate the integrals

a) [4 pt] $\int_0^{\frac{1}{3}} \frac{1}{\sqrt{4-9x^2}}\,dx$ **Answer:** $\pi/18$ or 10 degrees.

b) [5 pt] $\int \frac{1}{x(25+16\ln^2 x)}\,dx$ **Answer:** $\frac{1}{20}\arctan(\frac{4\ln x}{5})+C.$

c) [4 pt] $\int \arctan(\frac{x}{2})\,dx$ **Answer:** $x\arctan(\frac{x}{2})-\ln(x^2+4)+C$

d) [4 pt] $\int x\sin(1000x)\,dx$ **Answer:** $\frac{\sin(1000x)}{1,000,000}-\frac{x\cos(1000x)}{1000}+C$

e) [4 pt] $\int \sin^3 3x\,\cos^4 3x\,dx$ **Answer:** $\frac{\cos^7(3x)}{21}-\frac{\cos^5(3x)}{15}+C$

f) [4 pt] $\int \sec^5 x\tan^3 x\,dx$ **Answer:** $\frac{\sec^7(x)}{7}-\frac{\sec^5(x)}{5}+C$

g) [5 pt] $\int \frac{x^2}{\sqrt{4-x^2}}\,dx$ **Answer:** $2\arcsin(\frac{x}{2})-\frac{x\sqrt{4-x^2}}{2}+C$

h)[5 pt] $\int e^{3x}\cos(x)\,dx$ **Answer:** $e^{3x}\left(\frac{\sin x+3\cos x}{10}\right)+C$

**Bonus Problem**

Evaluate and check your result by differentiation:

$$\int (x^{-2}+x)(x^2+a^2)^{-1/2}\,dx$$

**Answer:** $\sqrt{x^2+a^2}-\frac{\sqrt{x^2+a^2}}{a^2x}+C$