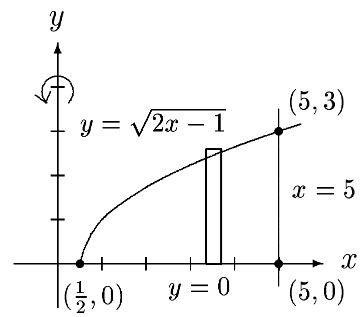
Math 1001 **Solutions** **Assignment #7**

1. (a) $y = \sqrt{2x-1}\,,\ y = 0\,,\ x = 5\,;$ about the y-axis

$$V = \int_a^b 2\pi r h\,dx = \int_{\frac{1}{2}}^{5} 2\pi x\sqrt{2x-1}\,dx \qquad \begin{array}{l} u = 2x-1 \\ du = 2dx \end{array} \qquad \begin{array}{l} x = 5 \Rightarrow u = 9 \\ x = \frac{1}{2} \Rightarrow u = 0 \end{array}$$

$$= 2\pi \int_0^9 \frac{u+1}{2}\cdot\sqrt{u}\cdot\frac{1}{2}\,du = \frac{\pi}{2}\int_0^9 (u^{3/2} + u^{1/2})\,du$$

$$= \frac{\pi}{2}\left(\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right)\Bigg|_0^9 = \frac{\pi}{2}\left(\frac{486}{5} + 18\right) = \frac{576\pi}{10} = \frac{288\pi}{5}$$

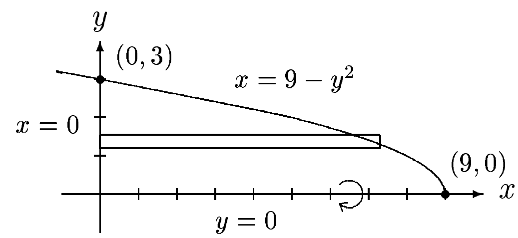


(b) $y = \sqrt{9-x}\,,\ y = 0\,,\ x = 0\,;$ about the x-axis

$$V = \int_a^b 2\pi r h\,dy = \int_0^3 2\pi y(9-y^2)\,dy$$

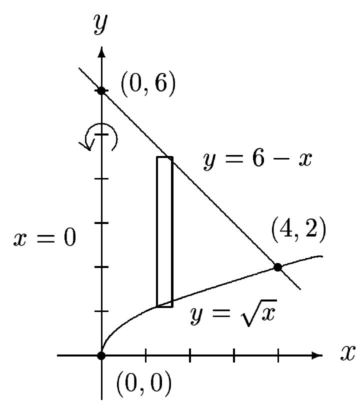
$$= 2\pi\int_0^3 (9y - y^3)\,dy = 2\pi\left(\frac{9y^2}{2} - \frac{y^4}{4}\right)\Bigg|_0^3$$

$$= 2\pi\left(\frac{81}{2} - \frac{81}{4}\right) = 2\pi\left(\frac{81}{4}\right) = \frac{81\pi}{2}$$



(c) $y = \sqrt{x}\,,\ x + y = 6\,,\ x = 0\,;$ about the y-axis

$$\begin{gathered} \sqrt{x} = 6 - x \\ x = 36 - 12x + x^2 \\ x^2 - 13x + 36 = 0 \\ (x-4)(x-9) = 0 \\ x = 4\,,\ x \neq 9 \\ (4, 2) \end{gathered}$$

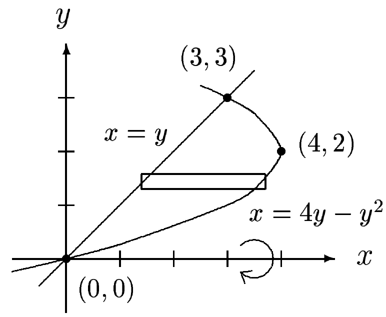


$$V = \int_a^b 2\pi r h\,dx = \int_0^4 2\pi x[(6-x) - \sqrt{x}\,]\,dx = 2\pi\int_0^4 (6x - x^2 - x\sqrt{x}\,)\,dx$$

$$= 2\pi\left(3x^2 - \frac{x^3}{3} - \frac{2}{5}x^{\frac{5}{2}}\right)\Bigg|_0^4 = 2\pi\left(48 - \frac{64}{3} - \frac{64}{5}\right) = \frac{416\pi}{15}$$

(d) $x = 4y - y^2\,,\ y = x\,;$ about the x-axis

$$4y - y^2 = y$$
$$y^2 - 3y = 0$$
$$y(y-3) = 0$$
$$y = 0,\ y = 3$$
$$(0,0), (3,3)$$



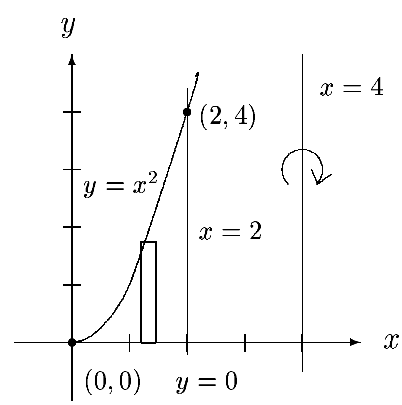
$$V = \int_a^b 2\pi r h\,dy = \int_0^3 2\pi y[(4y - y^2) - y]\,dy = 2\pi \int_0^3 y(3y - y^2)\,dy = 2\pi \int_0^3 (3y^2 - y^3)\,dy$$

$$= 2\pi \left(y^3 - \frac{y^4}{4} \right) \Bigg|_0^3 = 2\pi \left(27 - \frac{81}{4} \right) = 2\pi \left(\frac{27}{4} \right) = \frac{27\pi}{2}$$

(e) $x = \sqrt{y}\,,\ x = 2\,,\ y = 0\,;$ about the line $x = 4$

$$V = \int_a^b 2\pi r h\,dx = \int_0^2 2\pi (4 - x)x^2\,dx = 2\pi \int_0^2 (4x^2 - x^3)\,dx$$

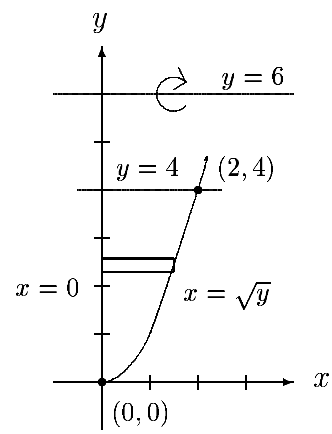
$$= 2\pi \left(\frac{4x^3}{3} - \frac{x^4}{4} \right) \Bigg|_0^2 = 2\pi \left(\frac{32}{3} - 4 \right) = \frac{40\pi}{3}$$



(f) $x = \sqrt{y}\,,\ x = 4\,,\ x = 0\,;$ about the line $y = 6$

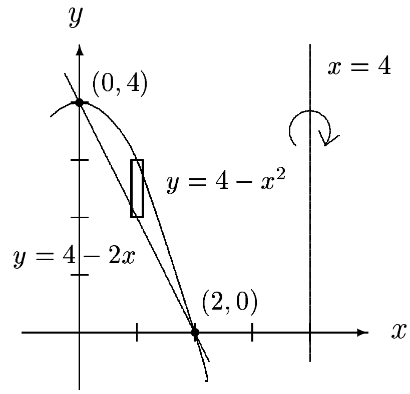
$$V = \int_a^b 2\pi r h\,dy = \int_0^4 2\pi (6 - y)\sqrt{y}\,dy = 2\pi \int_0^4 (6\sqrt{y} - y\sqrt{y}\,)\,dy$$

$$= 2\pi \left(4y^{\frac{3}{2}} - \frac{2}{5}\,y^{\frac{5}{2}} \right) \Bigg|_0^4 = 2\pi \left(32 - \frac{64}{5} \right) = \frac{192\pi}{5}$$



(g) $y = 4 - x^2\,,\ 2x + y = 4\,;$ about the line $x = 4$

$$4 - x^2 = 4 - 2x$$
$$x^2 - 2x = 0$$
$$x(x-2) = 0$$
$$x = 0,\ \ x = 2$$
$$(0,4), (2,0)$$

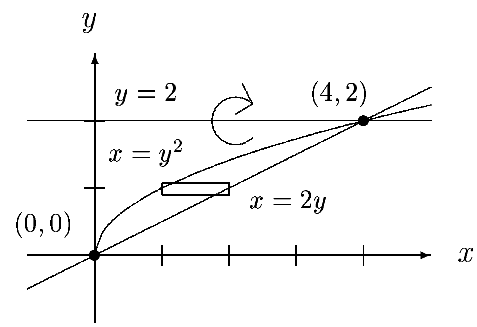


$$V = \int_a^b 2\pi r h\, dx = \int_0^2 2\pi(4-x)[(4-x^2)-(4-2x)]\, dx = 2\pi \int_0^2 (4-x)(2x-x^2)\, dx$$

$$= 2\pi \int_0^2 (x^3 - 6x^2 + 8x)\, dx = 2\pi \left(\frac{x^4}{4} - 2x^3 + 4x^2 \right) \Bigg|_0^2 = 2\pi(4 - 16 + 16) = 8\pi$$

(h) $y = \sqrt{x}\,,\ y = \frac{1}{2}x\,;$ about the line $y = 2$

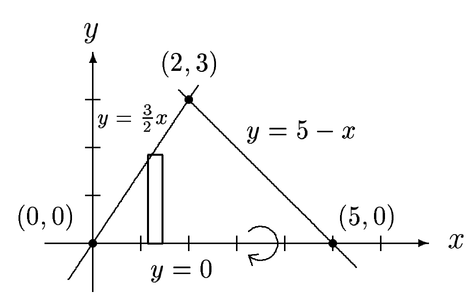
$$\sqrt{x} = \tfrac{1}{2}\,x$$
$$4x = x^2$$
$$x^2 - 4x = 0$$
$$x(x-4) = 0$$
$$x = 0\,,\ \ x = 4$$
$$(0,0)\,,\ \ (4,2)$$



$$V = \int_a^b 2\pi r h\, dy = \int_0^2 2\pi(2-y)(2y-y^2)\, dy = 2\pi \int_0^2 (4y - 4y^2 + y^3)\, dy$$
$$= 2\pi \left(2y^2 - \frac{4y^3}{3} + \frac{y^4}{4} \right) \Bigg|_0^2 = 2\pi \left(8 - \frac{32}{3} + 4 \right) = 2\pi \left(\frac{4}{3} \right) = \frac{8\pi}{3}$$

2. (a) $3x - 2y = 0\,,\ x + y = 5\,,\ y = 0\,;$ about the x-axis

$$\tfrac{3}{2}x = 5 - x$$
$$\tfrac{5}{2}x = 5$$
$$x = 2$$
$$(2,3)$$



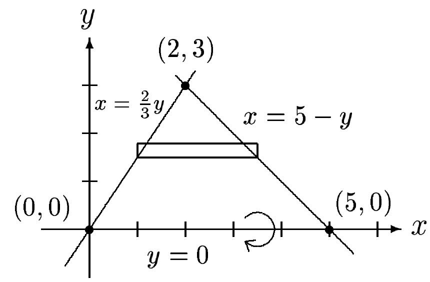
$$V = \int_a^b \pi R^2\, dx = \int_0^2 \pi(\tfrac{3}{2}x)^2\, dx + \int_2^5 \pi(5-x)^2 dx = \pi \int_0^2 \tfrac{9}{4}\,x^2\, dx + \pi \int_2^5 (5-x)^2\, dx$$

$$= \pi \left(\frac{3x^3}{4} \right) \Bigg|_0^2 - \pi \left[\frac{(5-x)^3}{3} \right] \Bigg|_2^5 = \pi(6-0) - \pi(0-9) = 6\pi + 9\pi = 15\pi$$

(b) $V = \int_a^b 2\pi r h\, dy = \int_0^3 2\pi y[(5-y) - \frac{2}{3}y]\, dy = 2\pi \int_0^3 y(5 - \frac{5}{3}y)\, dy$

$= 2\pi \int_0^3 (5y - \frac{5}{3}y^2)\, dy = 2\pi \left(\frac{5y^2}{2} - \frac{5y^3}{9}\right)\Bigg|_0^3$

$= 2\pi\left(\frac{45}{2} - 15\right) = 2\pi\left(\frac{15}{2}\right) = 15\pi$



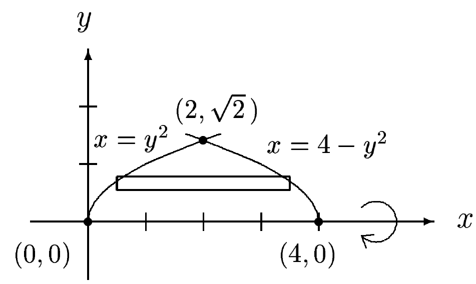
3. (a) $y = \sqrt{x}\,,\ y = \sqrt{4-x}\,,\ y = 0\,;$ about the x-axis

$\sqrt{x} = \sqrt{4-x}$

$x = 4 - x$

$x = 2$

$(2, \sqrt{2}\,)$



We use the shell method.

$V = \int_a^b 2\pi r h\, dy = \int_0^{\sqrt{2}} 2\pi y[(4-y^2) - y^2\,]\, dy = 2\pi \int_0^{\sqrt{2}} (4y - 2y^3)\, dy$

$= 2\pi\left(2y^2 - \frac{y^4}{2}\right)\Bigg|_0^{\sqrt{2}} = 2\pi(4-2) = 4\pi$

(b) $x + y = 3\,,\ y = \frac{1}{2}x\,,\ x = 0\,;$ about the line $x = 4$

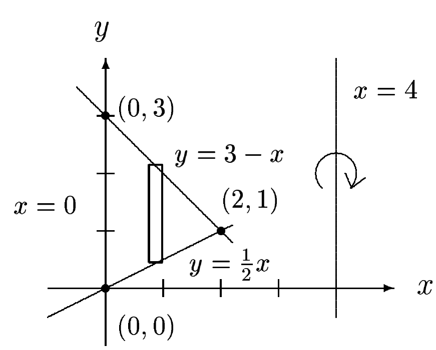
$3 - x = \frac{1}{2}x$

$6 - 2x = x$

$3x = 6$

$x = 2$

$(2, 1)$



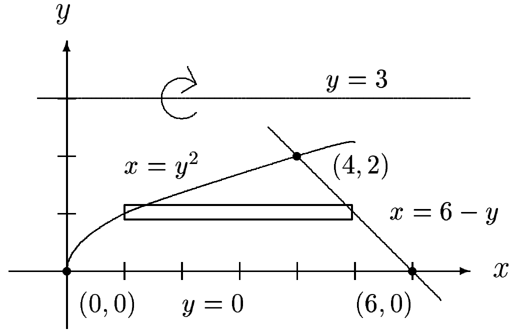
We use the shell method.

$V = \int_a^b 2\pi r h\, dx = \int_0^2 2\pi(4-x)\left[(3-x) - \frac{1}{2}x\right] dx = 2\pi \int_0^2 (4-x)\left(3 - \frac{3}{2}x\right) dx$

$= 2\pi \int_0^2 \left(12 - 9x + \frac{3}{2}x^2\right) dx = 2\pi\left(12x - \frac{9x^2}{2} + \frac{x^3}{2}\right)\Bigg|_0^2 = 2\pi(24 - 18 + 4) = 20\pi$

(c) $y = \sqrt{x}\,,\ x + y = 6,\ y = 0\,;$ about the line $y = 3$

$$\sqrt{x} = 6 - x$$
$$x = 36 - 12x + x^2$$
$$x^2 - 13x + 36 = 0$$
$$(x-4)(x-9) = 0$$
$$x = 4\,,\ x \neq 9$$
$$(4, 2)$$



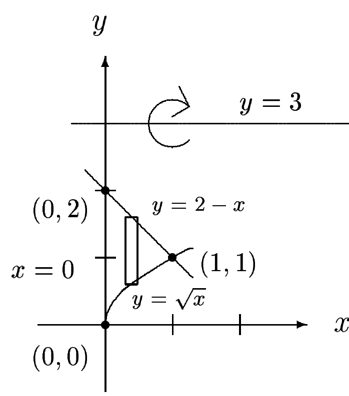
We use the shell method.

$$V = \int_a^b 2\pi r h\, dy = \int_0^2 2\pi(3-y)[(6-y) - y^2\,]\, dy = 2\pi \int_0^2 (18 - 9y - 2y^2 + y^3\,)\, dx$$

$$= 2\pi \left(18y - \frac{9y^2}{2} - \frac{2y^3}{3} + \frac{y^4}{4}\right)\Bigg|_0^2 = 2\pi\left(36 - 18 - \frac{16}{3} + 4\right) = 2\pi\left(\frac{50}{3}\right) = \frac{100\pi}{3}$$

(d) $y = \sqrt{x}\,,\ x + y = 2,\ x = 0\,;$ about the line $y = 3$

$$\sqrt{x} = 2 - x$$
$$x = 4 - 4x + x^2$$
$$x^2 - 5x + 4 = 0$$
$$(x-1)(x-4) = 0$$
$$x = 1\,,\ x \neq 4$$
$$(4, 2)$$



We use the washer method.

$$V = \int_a^b \pi(R^2 - r^2)\, dx = \int_0^1 \pi\left[(3 - \sqrt{x}\,)^2 - [3 - (2-x)]^2\right]\, dx = \int_0^1 \pi\left[(3 - \sqrt{x}\,)^2 - (1+x)^2\right]\, dx$$

$$= \pi \int_0^1 (9 - 6\sqrt{x} + x - 1 - 2x - x^2\,)\, dx = \pi \int_0^1 (8 - 6\sqrt{x} - x - x^2\,)\, dx$$

$$= \pi\left(8x - 4x^{\frac{3}{2}} - \frac{x^2}{2} - \frac{x^3}{3}\right)\Bigg|_0^1 = \pi\left(8 - 4 - \frac{1}{2} - \frac{1}{3}\right) = \frac{19\pi}{6}$$

4. (a) (i) $V = \int_0^4 \pi[2^2 - (\sqrt{x}\,)^2\,]\,dx$

(ii) $V = \int_0^2 2\pi y \cdot y^2\,dy$

(b) (i) $V = \int_0^2 \pi(y^2)^2\,dy$

(ii) $V = \int_0^4 2\pi x(2 - \sqrt{x}\,)\,dx$

(c) (i) $V = \int_0^4 \pi[(3 - \sqrt{x}\,)^2 - (3-2)^2\,]\,dx$

(ii) $V = \int_0^2 2\pi(3-y)y^2\,dy$

(d) (i) $V = \int_0^2 \pi[4^2 - (4 - y^2)^2\,]\,dy$

(ii) $V = \int_0^4 2\pi(4-x)(2 - \sqrt{x}\,)\,dx$