

1. (a)  $y = 2\sqrt{x}$ ,  $2x - 3y = 0$

$$2\sqrt{x} = \frac{2}{3}x$$

$$3\sqrt{x} = x$$

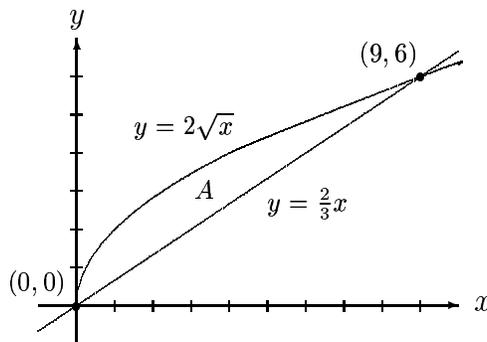
$$9x = x^2$$

$$x^2 - 9x = 0$$

$$x(x - 9) = 0$$

$$x = 0, x = 9$$

$$(0, 0), (9, 6)$$



$$A = \int_0^9 (2\sqrt{x} - \frac{2}{3}x) dx = \left( \frac{4}{3}x^{\frac{3}{2}} - \frac{1}{3}x^2 \right) \Big|_0^9 = (36 - 27) - (0 - 0) = 9$$

(b)  $y = 2x^2 + 1$ ,  $x + y = 2$

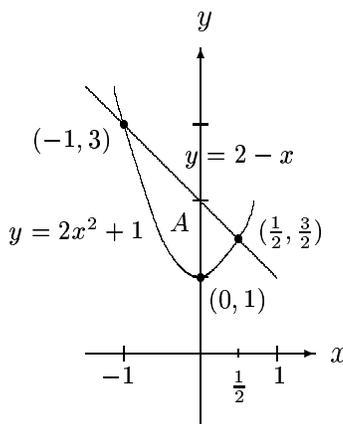
$$2x^2 + 1 = 2 - x$$

$$2x^2 + x - 1 = 0$$

$$(2x - 1)(x + 1) = 0$$

$$x = \frac{1}{2}, x = -1$$

$$\left(\frac{1}{2}, \frac{3}{2}\right), (-1, 3)$$



$$A = \int_{-1}^{\frac{1}{2}} [(2 - x) - (2x^2 + 1)] dx = \int_{-1}^{\frac{1}{2}} (1 - x - 2x^2) dx = \left( x - \frac{x^2}{2} - \frac{2x^3}{3} \right) \Big|_{-1}^{\frac{1}{2}}$$

$$= \left( \frac{1}{2} - \frac{1}{8} - \frac{1}{12} \right) - \left( -1 - \frac{1}{2} + \frac{2}{3} \right) = \frac{7}{24} - \left( -\frac{5}{6} \right) = \frac{27}{24} = \frac{9}{8}$$

(c)  $x = y^2 - 4$ ,  $x + y = 2$

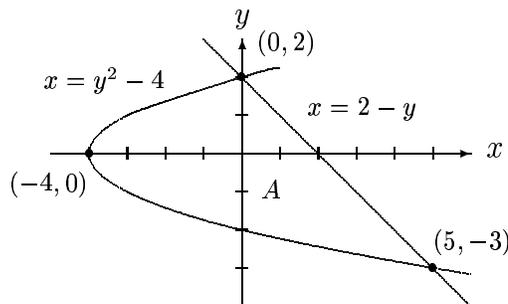
$$y^2 - 4 = 2 - y$$

$$y^2 + y - 6 = 0$$

$$(y + 3)(y - 2) = 0$$

$$y = -3, y = 2$$

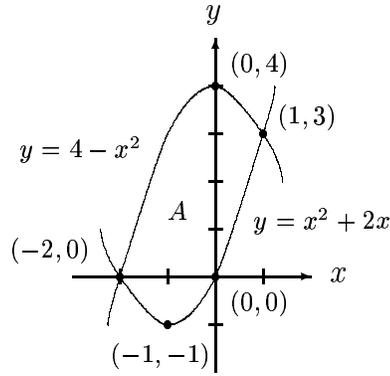
$$(5, -3), (0, 2)$$



$$A = \int_{-3}^2 [(2 - y) - (y^2 - 4)] dy = \int_{-3}^2 (6 - y - y^2) dy = \left( 6y - \frac{y^2}{2} - \frac{y^3}{3} \right) \Big|_{-3}^2$$

$$= \left( 12 - 2 - \frac{8}{3} \right) - \left( -18 - \frac{9}{2} + 9 \right) = \frac{22}{3} - \left( -\frac{27}{2} \right) = \frac{125}{6}$$

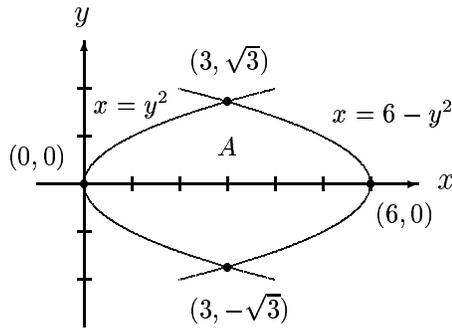
(d)  $y = x^2 + 2x, y = 4 - x^2$   
 $x^2 + 2x = 4 - x^2$   
 $2x^2 + 2x - 4 = 0$   
 $x^2 + x - 2 = 0$   
 $(x + 2)(x - 1) = 0$   
 $x = -2, x = 1$   
 $(-2, 0), (1, 3)$



$$A = \int_{-2}^1 [(4 - x^2) - (x^2 + 2x)] dx = \int_{-2}^1 (4 - 2x - 2x^2) dx = \left( 4x - x^2 - \frac{2x^3}{3} \right) \Big|_{-2}^1$$

$$= \left( 4 - 1 - \frac{2}{3} \right) - \left( -8 - 4 + \frac{16}{3} \right) = \frac{7}{3} - \left( -\frac{20}{3} \right) = \frac{27}{3} = 9$$

(e)  $x - y^2 = 0, x + y^2 = 6$   
 $y^2 = 6 - y^2$   
 $2y^2 = 6$   
 $y^2 = 3$   
 $y = \pm\sqrt{3}$   
 $(3, \sqrt{3}), (3, -\sqrt{3})$



$$A = \int_{-\sqrt{3}}^{\sqrt{3}} [(6 - y^2) - y^2] dy = \int_{-\sqrt{3}}^{\sqrt{3}} (6 - 2y^2) dy = \left( 6y - \frac{2y^3}{3} \right) \Big|_{-\sqrt{3}}^{\sqrt{3}}$$

$$= (6\sqrt{3} - 2\sqrt{3}) - (-6\sqrt{3} + 2\sqrt{3}) = 8\sqrt{3}$$

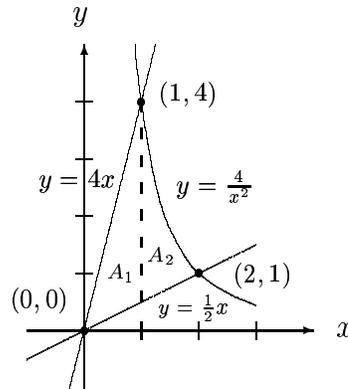
(f)  $y = \frac{4}{x^2}, y = 4x, y = \frac{1}{2}x$

$$\frac{4}{x^2} = 4x \quad \frac{4}{x^2} = \frac{1}{2}x \quad 4x = \frac{1}{2}x$$

$$x^3 = 1 \quad x^3 = 8 \quad x = 0$$

$$x = 1 \quad x = 2 \quad (0, 0)$$

$$(1, 4) \quad (2, 1)$$

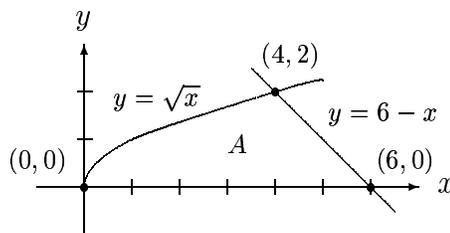


$$A = \int_0^1 \left( 4x - \frac{1}{2}x \right) dx + \int_1^2 \left( \frac{4}{x^2} - \frac{1}{2}x \right) dx = \int_0^1 \frac{7}{2}x dx + \int_1^2 \left( \frac{4}{x^2} - \frac{1}{2}x \right) dx$$

$$= \frac{7x^2}{4} \Big|_0^1 + \left( -\frac{4}{x} - \frac{x^2}{4} \right) \Big|_1^2 = \left( \frac{7}{4} - 0 \right) + \left[ (-2 - 1) - \left( -4 - \frac{1}{4} \right) \right] = \frac{7}{4} + \frac{5}{4} = \frac{12}{4} = 3$$

2.  $y = \sqrt{x}$ ,  $x + y = 6$ ,  $y = 0$

$$\begin{aligned} \sqrt{x} &= 6 - x \\ x &= 36 - 12x + x^2 \\ x^2 - 13x + 36 &= 0 \\ (x - 4)(x - 9) &= 0 \\ x &= 4, x \neq 9 \\ (4, 2) \end{aligned}$$

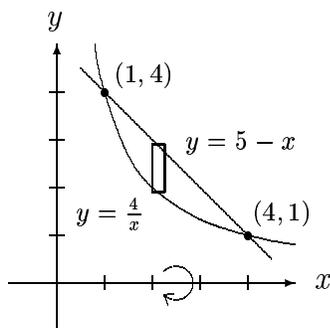


$$\begin{aligned} \text{(a)} \quad A &= \int_0^4 \sqrt{x} \, dx + \int_4^6 (6 - x) \, dx = \frac{2}{3} x^{\frac{3}{2}} \Big|_0^4 + \left( 6x - \frac{x^2}{2} \right) \Big|_4^6 = \left( \frac{16}{3} - 0 \right) + [(36 - 18) - (24 - 8)] \\ &= \frac{16}{3} + 2 = \frac{22}{3} \end{aligned}$$

$$\text{(b)} \quad A = \int_0^2 [(6 - y) - y^2] \, dy = \left( 6y - \frac{y^2}{2} - \frac{y^3}{3} \right) \Big|_0^2 = \left( 12 - 2 - \frac{8}{3} \right) - (0 - 0 - 0) = \frac{22}{3}$$

3. (a)  $x + y = 5$ ,  $xy = 4$ ;  $x$ -axis

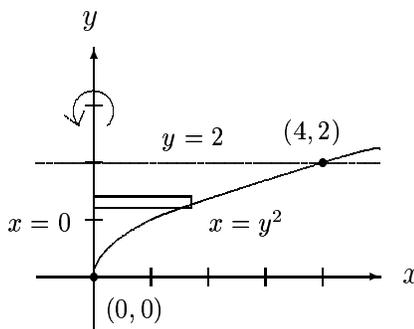
$$\begin{aligned} 5 - x &= \frac{4}{x} \\ x^2 - 5x + 4 &= 0 \\ (x - 1)(x - 4) &= 0 \\ x &= 1, x = 4 \\ (1, 4), (4, 1) \end{aligned}$$



$$\begin{aligned} V &= \int_a^b \pi(R^2 - r^2) \, dx = \int_1^4 \pi \left[ (5 - x)^2 - \left( \frac{4}{x} \right)^2 \right] dx = \pi \int_1^4 \left( 25 - 10x + x^2 - \frac{16}{x^2} \right) dx \\ &= \pi \left( 25x - 5x^2 + \frac{x^3}{3} + \frac{16}{x} \right) \Big|_1^4 = \pi [(100 - 80 + \frac{64}{3} + 4) - (25 - 5 + \frac{1}{3} + 16)] = 9\pi \end{aligned}$$

(b)  $y = \sqrt{x}$ ,  $y = 2$ ,  $x = 0$ ;  $y$ -axis

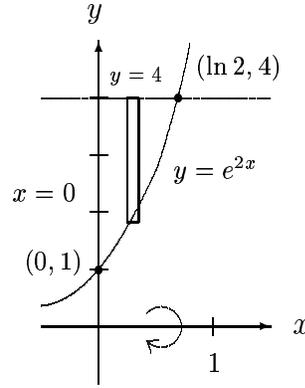
$$\begin{aligned} \sqrt{x} &= 2 \\ x &= 4 \\ (4, 2) \end{aligned}$$



$$V = \int_a^b \pi R^2 \, dy = \int_0^2 \pi (y^2)^2 \, dy = \pi \int_0^2 y^4 \, dy = \pi \frac{y^5}{5} \Big|_0^2 = \frac{32\pi}{5}$$

- (c)  $y = e^{2x}$ ,  $y = 4$ ,  $x = 0$ ;  $x$ -axis

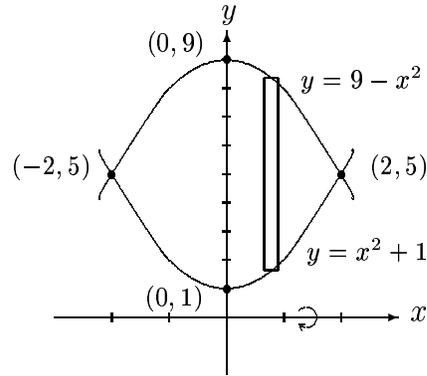
$$\begin{aligned} e^{2x} &= 4 \\ 2x &= \ln 4 \\ x &= \frac{1}{2} \ln 4 = \ln 2 \\ &(\ln 2, 4) \end{aligned}$$



$$\begin{aligned} V &= \int_a^b \pi(R^2 - r^2) dx = \int_0^{\ln 2} \pi[4^2 - (e^{2x})^2] dx = \pi \int_0^{\ln 2} (16 - e^{4x}) dx = \pi \left(16x - \frac{1}{4} e^{4x}\right) \Big|_0^{\ln 2} \\ &= \pi \left[(16 \ln 2 - 4) - \left(0 - \frac{1}{4}\right)\right] = \pi \left(16 \ln 2 - \frac{15}{4}\right) = \frac{(-15 + 64 \ln 2)\pi}{4} \end{aligned}$$

- (d)  $y = x^2 + 1$ ,  $y = 9 - x^2$ ;  $x$ -axis

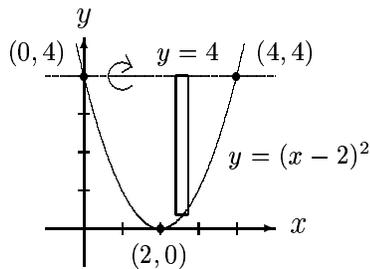
$$\begin{aligned} x^2 + 1 &= 9 - x^2 \\ 2x^2 &= 8 \\ x^2 &= 4 \\ x &= \pm 2 \\ &(2, 5), (-2, 5) \end{aligned}$$



$$\begin{aligned} V &= \int_a^b \pi(R^2 - r^2) dx = \int_{-2}^2 \pi[(9 - x^2)^2 - (x^2 + 1)^2] dx = \pi \int_{-2}^2 [(81 - 18x^2 + x^4) - (x^4 + 2x^2 + 1)] dx \\ &= \pi \int_{-2}^2 (80 - 20x^2) dx = 20\pi \int_{-2}^2 (4 - x^2) dx = 20\pi \left(4x - \frac{x^3}{3}\right) \Big|_{-2}^2 = 20\pi \left[\left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right)\right] = \frac{640\pi}{3} \end{aligned}$$

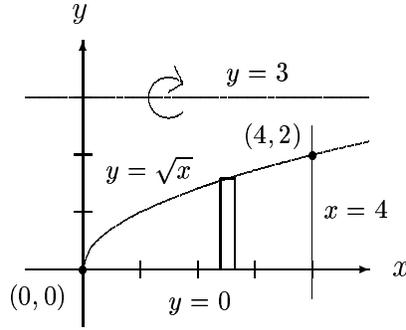
- (e)  $y = (x - 2)^2$ ,  $y = 4$ ;  $y = 4$

$$\begin{aligned} (x - 2)^2 &= 4 \\ x - 2 &= \pm 2 \\ x &= 2 \pm 2 \\ x &= 0, x = 4 \\ &(0, 4), (4, 4) \end{aligned}$$



$$\begin{aligned} V &= \int_a^b \pi R^2 dx = \int_0^4 \pi [4 - (x - 2)^2]^2 dx = \pi \int_0^4 [16 - 8(x - 2)^2 + (x - 2)^4] dx \\ &= \pi \left[16x - \frac{8(x - 2)^3}{3} + \frac{(x - 2)^5}{5}\right] \Big|_0^4 = \pi \left[\left(64 - \frac{64}{3} + \frac{32}{5}\right) - \left(0 + \frac{64}{3} - \frac{32}{5}\right)\right] = \frac{512\pi}{15} \end{aligned}$$

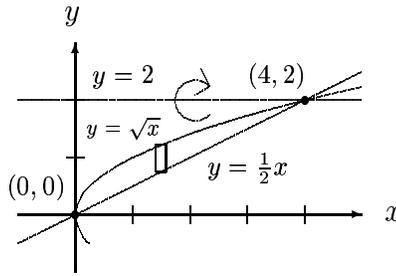
(f)  $y = \sqrt{x}$ ,  $y = 0$ ,  $x = 4$ ;  $y = 3$



$$\begin{aligned}
 V &= \int_a^b \pi(R^2 - r^2) dx = \int_0^4 \pi[3^2 - (3 - \sqrt{x})^2] dx = \pi \int_0^4 (9 - 9 + 6\sqrt{x} - x) dx \\
 &= \pi \int_0^4 (6\sqrt{x} - x) dx = \pi \left( 4x^{3/2} - \frac{x^2}{2} \right) \Big|_0^4 = \pi [(32 - 8) - (0 - 0)] = 24\pi
 \end{aligned}$$

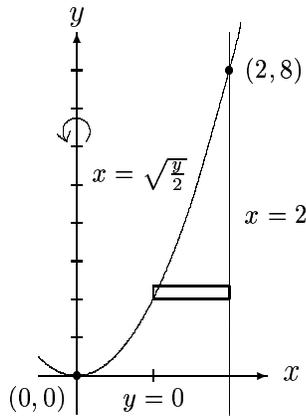
(g)  $x = y^2$ ,  $x = 2y$ ;  $y = 2$

$$\begin{aligned}
 y^2 &= 2y \\
 y^2 - 2y &= 0 \\
 y(y - 2) &= 0 \\
 y = 0, y &= 2 \\
 (0, 0), (4, 2)
 \end{aligned}$$



$$\begin{aligned}
 V &= \int_a^b \pi(R^2 - r^2) dx = \int_0^4 \pi \left[ \left( 2 - \frac{x}{2} \right)^2 - (2 - \sqrt{x})^2 \right] dx = \pi \int_0^4 \left[ \left( 4 - 2x + \frac{x^2}{4} \right) - (4 - 4\sqrt{x} + x) \right] dx \\
 &= \pi \int_0^4 \left( -3x + \frac{x^2}{4} + 4\sqrt{x} \right) dx = \pi \left( -\frac{3x^2}{2} + \frac{x^3}{12} + \frac{8x^{3/2}}{3} \right) \Big|_0^4 = \pi \left[ \left( -24 + \frac{16}{3} + \frac{64}{3} \right) - 0 \right] = \frac{8\pi}{3}
 \end{aligned}$$

(h)  $y = 2x^2$ ,  $y = 0$ ,  $x = 2$ ;  $y$ -axis



$$\begin{aligned}
 V &= \int_a^b \pi(R^2 - r^2) dy = \int_0^8 \pi \left[ 2^2 - \left( \sqrt{\frac{y}{2}} \right)^2 \right] dy = \pi \int_0^8 \left( 4 - \frac{y}{2} \right) dy = \pi \left( 4y - \frac{y^2}{4} \right) \Big|_0^8 \\
 &= \pi [(32 - 16) - (0 - 0)] = 16\pi
 \end{aligned}$$

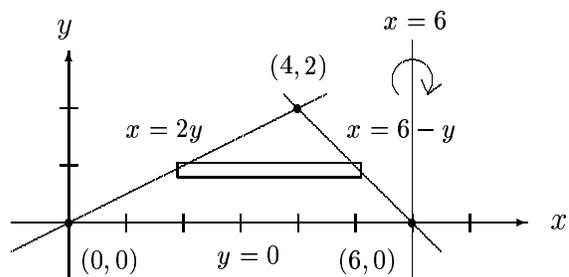
(i)  $y = \frac{1}{2}x$ ,  $x + y = 6$ ,  $y = 0$ ;  $x = 6$

$$\frac{1}{2}x = 6 - x$$

$$\frac{3}{2}x = 6$$

$$x = 4$$

$$(4, 2)$$



$$V = \int_a^b \pi (R^2 - r^2) dy = \int_0^2 \pi \{ (6 - 2y)^2 - [6 - (6 - y)]^2 \} dy$$

$$= \pi \int_0^2 [(36 - 24y + 4y^2) - y^2] dy = \pi \int_0^2 (36 - 24y + 3y^2) dy$$

$$= \pi (36y - 12y^2 + y^3) \Big|_0^2 = \pi [(72 - 48 + 8) - 0] = 32\pi$$