Math 1001 Solutions Assignment #1

1. (a) $\displaystyle\int \left(2t^2-1\right)^2\,dt = \int \left(4t^4-4t^2+1\right)dt = \frac{4}{5}t^5-\frac{4}{3}t^3+t+C$

(b) $\displaystyle\int \frac{x-4}{x\sqrt{x}}\,dx = \int \left(x^{-\frac{1}{2}}-4x^{-\frac{3}{2}}\right)dx = \frac{x^{\frac{1}{2}}}{\frac{1}{2}}-4\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}}+C = 2\sqrt{x}+\frac{8}{\sqrt{x}}+C$

(c)
$$\int \frac{(3x+1)^2}{\sqrt{x}}\,dx = \int \frac{9x^2+6x+1}{\sqrt{x}}\,dx = \int \left(9x^{\frac{3}{2}}+6x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right)dx = 9\,\frac{x^{\frac{5}{2}}}{\frac{5}{2}}+6\,\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+C$$
$$= \frac{18}{5}\,x^{\frac{5}{2}}+4x^{\frac{3}{2}}+2x^{\frac{1}{2}}+C = \frac{18}{5}\,x^2\sqrt{x}+4x\sqrt{x}+2\sqrt{x}+C$$

(d) $\displaystyle\int \frac{4x^2+3x+1}{x^2}\,dx = \int \left(4+\frac{3}{x}+\frac{1}{x^2}\right)dx = 4x+3\ln|x|-\frac{1}{x}+C$

(e) $\displaystyle\int \sec\theta(\tan\theta+\cos\theta)\,d\theta = \int (\sec\theta\tan\theta+1)\,d\theta = \sec\theta+\theta+C$

(f) $\displaystyle\int (3s+1)^8\,ds = \frac{1}{3}\,\frac{(3s+1)^9}{9}+C = \frac{(3s+1)^9}{27}+C$

(g) $\displaystyle\int \frac{1}{(2x-5)^4}\,dx = \int (2x-5)^{-4}\,dx = \frac{1}{2}\,\frac{(2x-5)^{-3}}{-3}+C = -\frac{1}{6(2x-5)^3}+C$

(h) $\displaystyle\int \frac{8}{4x+1}\,dx = 8\cdot\frac{1}{4}\ln|4x+1|+C = 2\ln|4x+1|+C$

(i)
$$\int \frac{(e^x+1)^3}{2e^x}\,dx = \int \frac{e^{3x}+3e^{2x}+3e^x+1}{2e^x}\,dx = \int \left(\frac{1}{2}e^{2x}+\frac{3}{2}e^x+\frac{3}{2}+\frac{1}{2}e^{-x}\right)dx$$
$$= \frac{1}{4}e^{2x}+\frac{3}{2}e^x+\frac{3}{2}x-\frac{1}{2}e^{-x}+C$$

(j) $\displaystyle\int (14\sin 2x-9\cos 3x)\,dx = 14\cdot\frac{1}{2}(-\cos 2x)+9\cdot\frac{1}{3}\sin 3x+C = -7\cos 2x-3\sin 3x+C$

(k) $\displaystyle\int \frac{6}{\cos^2 2x}\,dx = \int 6\sec^2 2x\,dx = 6\cdot\frac{1}{2}\tan 2x+C = 3\tan 2x+C$

2. (a) $\displaystyle f'(x) = \frac{x^2-1}{x^5}\ , \quad f(2)=3$

$$f(x) = \int \frac{x^2-1}{x^5}\,dx = \int \left(x^{-3}-x^{-5}\right)dx = \frac{x^{-2}}{-2}-\frac{x^{-4}}{-4}+C = -\frac{1}{2x^2}+\frac{1}{4x^4}+C$$

$$f(2)=3 \;\Rightarrow\; -\frac{1}{8}+\frac{1}{64}+C=3 \;\Rightarrow\; C=\frac{199}{64}$$

$$f(x) = -\frac{1}{2x^2}+\frac{1}{4x^4}+\frac{199}{64}$$

(b) $\dfrac{dy}{dx} = \dfrac{3}{\sqrt{2x+1}}$, $\quad y(4) = -7$

$$y = \int \frac{3}{\sqrt{2x+1}}\,dx = \int 3(2x+1)^{-\frac{1}{2}}\,dx = 3 \cdot \frac{1}{2}\,\frac{(2x+1)^{\frac{1}{2}}}{\frac{1}{2}} + C = 3\sqrt{2x+1} + C$$

$$y(4) = -7 \;\Rightarrow\; 3\sqrt{9} + c = -7 \;\Rightarrow\; c = -16$$

$$y = 3\sqrt{2x+1} - 16$$

(c) $y'' = \dfrac{1}{2x^2}$, $\quad y(1) = 2\,,\; y'(1) = 3$

$$y' = \int \frac{1}{2x^2}\,dx = \int \frac{1}{2}x^{-2}\,dx = \frac{1}{2}\,\frac{x^{-1}}{-1} + C = -\frac{1}{2x} + C$$

$$y'(1) = 3 \;\Rightarrow\; -\frac{1}{2} + C = 3 \;\Rightarrow\; C = \frac{7}{2}$$

$$y' = -\frac{1}{2x} + \frac{7}{2}$$

$$y = \int \left(-\frac{1}{2x} + \frac{7}{2}\right) dx = -\frac{1}{2}\ln|x| + \frac{7}{2}\,x + C$$

$$y(1) = 2 \;\Rightarrow\; -0 + \frac{7}{2} + C = 2 \;\Rightarrow\; C = -\frac{3}{2}$$

$$y = -\frac{1}{2}\ln|x| + \frac{7}{2}\,x - \frac{3}{2}$$

(d) $f''(x) = 3e^x + 5\sin x$, $\quad f(0) = 1\,,\; f'(0) = 2$

$$f'(x) = \int (3e^x + 5\sin x)\,dx = 3e^x - 5\cos x + C$$

$$f'(0) = 2 \;\Rightarrow\; 3 - 5 + C = 2 \;\Rightarrow\; C = 4$$

$$f'(x) = 3e^x - 5\cos x + 4$$

$$f(x) = \int (3e^x - 5\cos x + 4)\,dx = 3e^x - 5\sin x + 4x + C$$

$$f(0) = 1 \;\Rightarrow\; 3 - 0 + 0 + C = 1 \;\Rightarrow\; C = -2$$

$$f(x) = 3e^x - 5\sin x + 4x - 2$$

3. Let the positive direction be upward and s be the distance from the ground. Then

$$a(t) = -32$$

$$v(t) = \int a(t)\,dt = \int -32\,dt = -32t + C$$

$$v(0) = 320 \;\Rightarrow\; -0 + c = 320 \;\Rightarrow\; C = 320$$

$$v(t) = -32t + 320$$

$$s(t) = \int v(t)\,dt = \int (-32t + 320)\,dt = -16t^2 + 320t + C$$

$$s(0) = 0 \;\Rightarrow\; -0 + 0 + C = 0 \;\Rightarrow\; C = 0$$

$$s(t) = -16t^2 + 320t \qquad \text{[or start with } s(t) = \tfrac{1}{2}at^2 + v_0 t + s_0\,]$$

The ball reaches its maximum height when

$$v(t) = 0$$

$$-32t + 320 = 0$$

$$t = 10 \text{ seconds}$$

and the maximum height is

$$s(10) = -16(10)^2 + 320(10) = -1600 + 3200 = 1600 \text{ ft}$$

4. Let $s(t)$ be the distance taveled after the brakes are applied, $t = 0$ at the instant the brakes are applied, and the positive direction be the direction of motion. Then

$$a(t) = -40$$

$$v(t) = \int a(t)\,dt = \int -40\,dt = -40t + C$$

$$v(0) = 88 \;\Rightarrow\; -0 + C = 88 \;\Rightarrow\; c = 88$$

$$v(t) = -40t + 88$$

$$s(t) = \int v(t)\,dt = \int (-40t + 88)\,dt = -20t^2 + 88t + C$$

$$s(0) = 0 \;\Rightarrow\; -0 + 0 + C = 0 \;\Rightarrow\; C = 0$$

$$s(t) = -20t^2 + 88t$$

The car comes to rest when

$$v(t) = 0$$

$$-40t + 88 = 0$$

$$t = \tfrac{88}{40} = \tfrac{11}{5} \text{ seconds}$$

Then the distance the car travels before coming to rest is

$$s(\tfrac{11}{5}) = -20\left(\tfrac{11}{5}\right)^2 + 88\left(\tfrac{11}{5}\right) = -\frac{484}{5} + \frac{968}{5} = \frac{484}{5} \text{ ft} \quad \text{or } 96.8 \text{ ft}$$

5. (a) $\frac{1}{3}+\frac{1}{5}+\frac{1}{9}+\frac{1}{17}+\frac{1}{33}+\frac{1}{65}=\sum_{i=1}^{6}\frac{1}{2^i+1}$

(b) $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}=\sum_{i=1}^{5}\frac{(-1)^{i+1}}{i}$

6. (a) $\sum_{j=1}^{12}(2j-1)^2=\sum_{j=1}^{12}(4j^2-4j+1)=4\sum_{j=1}^{12}j^2-4\sum_{j=1}^{12}j+\sum_{j=1}^{12}1=4\cdot\frac{12(13)(25)}{6}-4\cdot\frac{12(13)}{2}-$

$$=2600-312+12=2300$$

(b) $\sum_{i=1}^{10}i(3i^2+5)=\sum_{i=1}^{10}(3i^3+5i)=3\sum_{i=1}^{10}i^3+5\sum_{i=1}^{10}i=3\cdot\frac{10^2 11^2}{4}+5\cdot\frac{10(11)}{2}=9075+275=$ 9350

7. $S_n=\sum_{i=1}^{n}\left(1+\frac{2i}{n}\right)^2\left(\frac{2}{n}\right)=\sum_{i=1}^{n}\left(1+\frac{4i}{n}+\frac{4i^2}{n^2}\right)\frac{2}{n}=\sum_{i=1}^{n}\left(\frac{2}{n}+\frac{8i}{n^2}+\frac{8i^2}{n^3}\right)$

$$=\frac{2}{n}\sum_{i=1}^{n}1+\frac{8}{n^2}\sum_{i=1}^{n}i+\frac{8}{n^3}\sum_{i=1}^{n}i^2=\frac{2}{n}\cdot n+\frac{8}{n^2}\cdot\frac{n(n+1)}{2}+\frac{8}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}$$

$$=2+\frac{4(n+1)}{n}+\frac{4(n+1)(2n+1)}{3n^2}$$

And then

$$\lim_{n\to\infty}S_n=\lim_{n\to\infty}\left[2+\frac{4(n+1)}{n}+\frac{4(n+1)(2n+1)}{3n^2}\right]=2+4+\frac{8}{3}=\frac{26}{3}$$

8. (a) $f(x)=3x-4\geq 0$ and continuous on $[2,5]$.

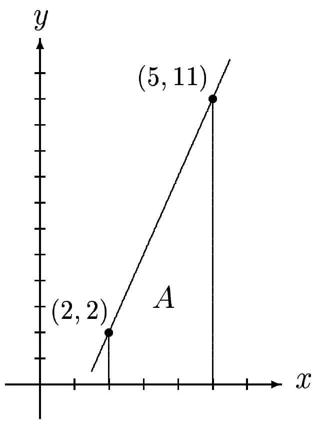
$$A=\lim_{n\to\infty}\sum_{i=1}^{n}f(x_i)\triangle x \qquad \triangle x=\frac{b-a}{n}=\frac{5-2}{n}=\frac{3}{n},\ x_i=a+i\triangle x=2+i\left(\frac{3}{n}\right)=2+\frac{3i}{n}$$

$$=\lim_{n\to\infty}\sum_{i=1}^{n}f\left(2+\frac{3i}{n}\right)\cdot\frac{3}{n}=\lim_{n\to\infty}\sum_{i=1}^{n}\left[3\left(2+\frac{3i}{n}\right)-4\right]\cdot\frac{3}{n}$$

$$=\lim_{n\to\infty}\sum_{i=1}^{n}\left[2+\frac{9i}{n}\right]\cdot\frac{3}{n}=\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{6}{n}+\frac{27i}{n^2}\right)$$

$$=\lim_{n\to\infty}\left[\frac{6}{n}\sum_{i=1}^{n}1+\frac{27}{n^2}\sum_{i=1}^{n}i\right]=\lim_{n\to\infty}\left[\frac{6}{n}\cdot n+\frac{27}{n^2}\cdot\frac{n(n+1)}{2}\right]$$

$$=6+\frac{27}{2}=\frac{39}{2}$$



(b) $f(x) = 4 - x^2 \ge 0$ and continuous on $[-1, 1]$.

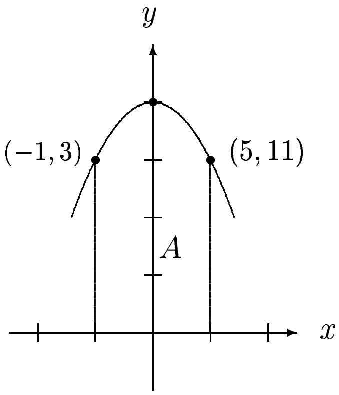
$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\triangle x \qquad \triangle x = \frac{b-a}{n} = \frac{1-(-1)}{n} = \frac{2}{n},\; x_i = a + i\triangle x = -1 + i\left(\frac{2}{n}\right) = -1 + \frac{2i}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(-1 + \frac{2i}{n}\right)\cdot\frac{2}{n} = \lim_{n\to\infty} \sum_{i=1}^{n} \left[4 - \left(-1 + \frac{2i}{n}\right)^2\right]\cdot\frac{2}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[4 - \left(1 - \frac{4i}{n} + \frac{4i^2}{n^2}\right)\right]\cdot\frac{2}{n} = \lim_{n\to\infty} \sum_{i=1}^{n} \left(3 + \frac{4i}{n} - \frac{4i^2}{n^2}\right)\cdot\frac{2}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{6}{n} + \frac{8i}{n^2} - \frac{8i^2}{n^3}\right) = \lim_{n\to\infty} \left[\frac{6}{n}\sum_{i=1}^{n} 1 + \frac{8}{n^2}\sum_{i=1}^{n} i - \frac{8}{n^3}\sum_{i=1}^{n} i^2\right]$$

$$= \lim_{n\to\infty} \left[\frac{6}{n}\cdot n + \frac{8}{n^2}\cdot\frac{n(n+1)}{2} - \frac{8}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}\right]$$

$$= 6 + 4 - \frac{8}{3} = \frac{22}{3}$$