

Mathematics 2130

Lab 2012W–4A

Sequences and Switches

Consider a binary sequence consisting of m zeroes and n ones in any order. We call an element in the sequence a “switch” if it differs from the previous element. For instance, if $m = n = 3$, the sequence $\{0, 0, 1, 0, 1, 1\}$ has three switches: the third, fourth and fifth elements.

Within this simple framework, we can ask many questions. Most basically, if we ignore switches altogether, what is the total number of binary sequences with m zeroes and n ones? What is the minimum number of switches such a sequence can have? What is the maximum number? Most importantly, considering all possible sequences, what is the *average* number of switches?

Of course, there is nothing really special about binary sequences. A k -ary sequence is one in which every element is an integer from $0, 1, \dots, k-1$. For example, $\{2, 2, 0, 1, 1, 3, 3, 3, 2, 3\}$ is a 4-ary sequence (for which the third, fourth, sixth, ninth and tenth elements are switches). What can you say about switches when you consider k -ary sequences?

Prepare a report that addresses the issues mentioned above. As always, supporting mathematical reasoning is essential.