Mathematics 2130
Lab 2012W–3
The Slapshot Curve

Bob is a minor hockey coach. There is a big tournament coming up, and he figures that his young team has a good chance of making it to the playoffs. This would be a great boost to his team's morale.

On Bob's team is a young player named Tina, a.k.a. "Slapshot". She earned her nickname by being recognized as the kid with the fastest slapshot this league has ever seen. However, Tina has two little problems: her accuracy with the puck is poor, and she can't skate well yet. As a coach, Bob wants to capitalize on Tina's skills as best as he can. For this tournament, his advice to Tina is the following:

> Whenever she gets the puck, she is to skate up the ice in a straight line (parallel to the sides) from wherever she is, and shoot when she reaches the point where the angle on the net is largest.

This way, her problem with inaccuracy is minimized and the skating is as simple as it can be. The problem is, Bob can't figure out how far up the ice Tina should skate in order to have the largest possible angle on the net. This is where you come in. Your task is to solve this problem for Tina (and Bob).

The solution set of this problem is the locus of points to which Tina should skate. Let's call this the *slapshot curve*. You should discover everything you can about the slapshot curve; it is generated by an optimization problem of a kind you should be familiar with from calculus.

To begin, explore this problem numerically. Use the computer algebra software **Maple** to numerically determine the points on this curve. You may find the commands `diff` and `solve` useful, at least as starting points. In addition to the numerical solution, you should also explore theoretical solutions using differential calculus.

In terms of the physical problem, there are a few things to keep in mind. First, tournament rules stipulate that Tina is not allowed to enter the goal crease, so if Tina is headed straight at the net, she cannot go all the way up to the goal line. As well, Bob coaches soccer in the summertime and has realized that whatever works for ice hockey ought to be adaptable for soccer too. Make your solutions as general as possible.

When your analysis and computer programs are completed, prepare a report providing a full description of your solution. All key terms should be clearly defined. Supporting graphics illustrating the geometric quantities of interest should be included as well. Supporting mathematical reasoning is essential. This, combined with a few truly informative illustrations, is all that is required.