Training programming exercises for the 2120 Final Exam

1. Factoring a cubic polynomial

   Given a cubic polynomial $P(x) = ax^3 + bx^2 + cx + d$ with integer coefficients, determine whether it has an integer root by trying all divisors of d. If $P(k) = 0$ for some divisor k of d, the factor the $P(x)$ as follows:

   $$ax^3 + bx^2 + cx + d = (x - k)(ax^2 + b'x + c') \quad \Rightarrow \quad c'k = -d, \quad b'k = -d + c'.$$

   Then find and print all real roots of $P(x)$ (one root is the k you'd find; others may come from the factored out quadratic polynomial if it has real roots). In this question, an array is not needed; this is just an exercise on loops, if/else, and the mod (%) operator.

2. Evaluation of a trigonometric polynomial

   Given an integer N, the set of coefficients $a_0, \dots a_N$, and a real number θ, evaluate the trigonometric polynomial

   $$T(\theta) = a_0 + a_1 \cos\theta + a_2 \cos(2\theta) + \cdots + a_N \cos(N\theta).$$

   (User interface considerations: it is likely the user will normally want to evaluate the same trigonometric polynomial at more than one point. Therefore it would be nice to provide a possibility not to re-type all the coefficients anew for every single evaluation.)

3. Diameter of a point set

   A set of N points in the plane is given; each point P_i is characterized by its two coordinates (x_i, y_i). Determine two points for which the distance

   $$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

   is maximum. Print the maximum distance and the indices i_{min} and j_{min} of the most distant points. If there is more than one pair of points for which the maximum is attained, find and print any one such pair.

4. Base 10 to base b conversion

   Given two integers, $a \geq 0$ and $b \geq 2$, find the representation of a in base b:

   $$a = a_0 + a_1 b + \cdots + a_k b^k.$$

   > A sketch of the algorithm:
   > Find a_0 as the reminder in division of a by b. Then divide a by b (as intergers, throwing away the fractional part). The quotient will have the base b representation
   >
   > $$[a/b] = a_1 + a_2 b \cdots + a_k b^{k-1}.$$
   >
   > Apply the same procedure to find a_1 and reduce the quotient further. Continue until the quotient becomes 0.

   Why is an array needed in a proper version of a program? Because we calculate the digits of the base b representation in the *reverse* order. We need to store them and then print them backwards.