MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

FINAL EXAM | **Pure Mathematics 3370** | FALL 2005

Marks

[2] 1. (a) Find the inverse of 97 modulo 192.

[3] (b) Find all the incongruent solutions of the congruence $485x \equiv 5 \pmod{960}$.

[2] (c) Solve the Diophantine equation $192x + 97y = 5000$.

[2] (d) Find the positive solutions, if any.

[2] (e) Find the smallest positive solution of the Diophantine equation $192x - 97y = 5000$.

[2] (f) Given $n = 221 = 17 \times 13, e = 97$, and the encryption function $E : M \mapsto M^e \pmod{n}$, find d so that $D : C \mapsto C^d \pmod{n}$ is the decryption function in the RSA–Algorithm. (That is, $D \circ E =$ the identity function for integers mod n which are relatively prime to n.)

[3] 2. If $c \mid ab$ and $(b, c) = 1$, prove that $c \mid a$.

[3] 3. Let $\{f_n\}$ be the Fibonacci sequence. For $n \geq 1$ prove, by mathematical induction, that $f_n = \dfrac{\alpha^n - \beta^n}{\sqrt{5}}$, where α, β are the roots of $x^2 - x - 1 = 0$, α being the larger root.

[4] 4. (a) State and prove Euler's Theorem.

[3] (b) Find the remainder when 11^{348} is divided by 54.

[3] 5. (a) Find x which satisfy simultaneously $x \equiv -3 \pmod{12}$ $x \equiv 1 \pmod{5}$ and $x \equiv 14 \pmod{17}$.

[2] (b) Use the Chinese Remainder Theorem to find the last two digits of the number 2^{1000}.

[2] 6. (a) Define the *order* of an integer modulo a positive integer m.

[3] (b) If a has order h modulo m and b is the inverse of a modulo m, prove that b also has order h. (Note $ab \equiv 1 \pmod{m}$.)

[2] (c) Calculate $\phi(\phi(100 \times 19^3))$, where ϕ is Euler's phi function.

[3] 7. Find all the primitive Pythagorean triples a, b, c with $a^2 + b^2 = c^2$ where one of a, b, c is equal to 140.

[3] 8. (a) Factor into Gaussian primes the number $27300(1 + 3i)$.

[3] (b) State and prove the Division Algorithm for Gaussian Integers.

PLEASE TURN OVER

[3] 9. Do **ONE** part only:

(a) State and prove Wilson's Theorem.

(b) Euclid defined perfect numbers and discovered a formula for even perfect numbers. Euler, 2000 years later, proved that this formula gave *all* the even perfect numbers. State clearly one of these results and prove it.

[50]