PMAT 4340 – Combinatorial Analysis   Assignment #8
Fall 2003

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at: 3:00 pm on Wednesday November 26th.

1. Assuming that n is a power of 2, solve the following recurrence relations:

   (a) $a_n = a_{\frac{n}{2}} + 7$, $a_1 = 5$.

   (b) $a_n = 4a_{\frac{n}{2}} - 5n$, $a_1 = 2$.

   (c) $a_n = -3a_{\frac{n}{2}} + 2n$, $a_1 = 1$.

2. Assuming that n is a power of k, a_1 is given, and that $c > 0$, prove that $a_n = An^2 + Bn^{\log_k c}$ is a solution to the recurrence relation $a_n = ca_{\frac{n}{k}} + dn^2$, where A and B are constants.

3. Solve the following linear recurrence relation: $a_n = -7a_{n-1} - 9a_{n-2}$, $a_0 = 0, a_1 = 1$.

4. Solve the following inhomogeneous recurrence relations:

   (a) $a_n = 2a_{n-1} + n$, $a_0 = 17$.

   (b) $a_n = 4a_{n-1} - 3^n$, $a_0 = 1$.

   (c) $a_n = 3a_{n-1} - 2n + n^2$, $a_0 = 0$.

5. Use generating functions to solve the following recurrence relations:

   (a) $a_n = a_{n-1} - 3n$, $a_0 = 3$.

   (b) $a_n = 3a_{n-1} + 4a_{n-2}$, $a_0 = 1, a_1 = 2$.

   (c) $a_n = 3a_{n-1} + 2^n$, $a_0 = 1$.

   (d) $a_n = 3a_{n-1} - 2a_{n-2} + n$, $a_0 = 2, a_1 = 4$.

6. Exercise 7.5.14.

7. What is the size of the set of symmetries of a cube? Explain.

8. Exercise 9.1.16.