## PMAT 4340 – Combinatorial Analysis Fall 2003

Assignment #5

## Instructions

- Answer each question completely; justify your answers.
- This assignment is due at: 3:00 pm on Wednesday October 15th.
- 1. Suppose that Nestle has contracted with an airline to provide small packages of Smarties that are to distributed to the airline's passengers while in flight. Each package is to contain 20 individual Smarties, coloured red, blue, green, and yellow. Colours red, green, and yellow each need to be represented at least twice per package, but blue should occur at most 3 times (thereby attempting to design the blue candies to be rare).
  - (a) How many different ways are there to fill each package, subject to these constraints?
  - (b) What fraction of the possible ways to fill each package contain blue Smarties?
- 2. For each integer  $r \ge 0$ , find  $[x^r]G(x)$ , where  $G(x) = \left(\frac{1}{1-x^2}\right)\left(\frac{2}{1-x^3}\right)$ .
- 3. Let  $a_r$  denote the number of ways to express r as a sum of perfect squares of positive integers.
  - (a) Find a generating function for  $a_r$ .
  - (b) Determine  $a_{30}$ .
- 4. Find a generating function for  $a_r$ , the number of integer solutions to the equation  $e_1 + 3e_2 + 3e_3 + 7e_4 = r$ , where  $0 \le e_1$ ,  $0 \le e_2$ ,  $2 \le e_3 \le 8$ , and  $0 \le e_4 \le 20$ .
- 5. Find a generating function for  $a_r$ , the number of partitions of r into 4 parts.
- 6. Exercise 6.3.16.
- 7. Exercise 6.3.17.