PMAT 4340 – Combinatorial Analysis   Assignment #4
Fall 2003

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at: 3:00 pm on Monday October 6th.

1. Exercise 5.5.28.

2. Exercise 5.5.32.

3. Let a_r denote the number of ways to distribute r identical seeds to a dozen farmers, such that Farmer Brown gets at least 150 seeds and each remaining farmer gets at least 50 seeds, and such that Old MacDonald gets no more than 1000 seeds. Find a generating function for a_r.

4. Find a generating function for the number of ways to withdraw \$150 from an ATM that dispenses all denominations of paper currency that are in circulation. What coefficient would be needed in order to solve the problem?

5. Find a generating function for the number of ways to select 5 cards, one each from 5 decks of cards, such that

   (a) exactly r aces are selected.
   (b) exactly r spades are selected.
   (c) exactly r of the cards are the ace of spades.

   Solve each of these problems for the case in which $r = 2$.

6. Find a generating function for the number of selections of r coins that can be made, given that coins are only available to be selected in rolls (of 50 pennies, 50 nickels, 50 dimes, 50 quarters, and 20 loonies).

7. Find $[x^5]G(x)$, where

   (a) $G(x) = (1 - x)^{-9}$.
   (b) $G(x) = (1 - x^2)^{-1}$.
   (c) $G(x) = (1 + x)^{-5}$.

8. Find $[x^8]G(x)$, where $G(x) = (1 + x + x^2 + \cdots)^2 (x^2 + x^3 + \cdots + x^{99})^3$.

9. Find $[x^{10}]G(x)$, where $G(x) = (1 + x + x^2)^{20}$.

10. Use a generating function to determine the number of integers x, $0 \le x < 1000$, whose digits sum to 15.

11. Find $[x^r]G(x)$, where $G(x) = \left(\sum_{k \ge 0} k^2 x^k \right) \left(\sum_{k \ge 0} (k + 1)x^k \right)$.