PMAT 3331 – Projective Geometry — Assignment #4
Winter 2003

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 3:00 pm on February 19, 2003.
- Please place your completed assignment in Box 35.

1. The 4 axioms for projective planes are:

   i. Every pair of distinct points are incident with exactly one line.

   ii. Every pair of distinct lines intersect in a single point.

   iii. There are at least 3 distinct points on every line.

   iv. Each point is on at least 3 distinct lines.

   For each integer $n \geq 2$, construct a set $\mathcal{P}$ of points and a set $\mathcal{L}$ of lines such that $|\mathcal{P}| = n$, axioms 1 and 2 are satisfied, but both of axioms 3 and 4 are violated.

2. Consider the points $(4, 5)$ and $(-2, 2)$ in $\mathbb{R}^2$.

   (a) What points in $P_2(\mathbb{R})$ do these points correspond to?

   (b) What line in $P_2(\mathbb{R})$ contains both of the corresponding points?

3. What line of $P_2(\mathbb{R})$ does the line $y = 3x - 2$ of $\mathbb{R}^2$ correspond to? Sketch this line in the projective plane, illustrating its points of intersection with ℓ_1, ℓ_2, and ℓ_∞.

4. Consider the lines $y = 3x + 4$ and $y = 4x - 8$ in $\mathbb{R}^2$. Determine the corresponding lines in $P_2(\mathbb{R})$ and their point of intersection. Sketch these lines in $P_2(\mathbb{R})$ and identify all points of interest.

5. Let k be fixed. Prove that the vertical line $x = k$ in $\mathbb{R}^2$ corresponds to a line in $P_2(\mathbb{R})$. What is this corresponding line?

6. (a) Let m, b_1, and b_2 be fixed such that $b_1 \neq b_2$. The lines $y = mx + b_1$ and $y = mx + b_2$ in $\mathbb{R}^2$ correspond to two lines in $P_2(\mathbb{R})$. What is the point of intersection of these two lines in $P_2(\mathbb{R})$?

   (b) Let k_1 and k_2 be fixed such that $k_1 \neq k_2$. The lines $x = k_1$ and $x = k_2$ in $\mathbb{R}^2$ correspond to two lines in $P_2(\mathbb{R})$. What is the point of intersection of these two lines in $P_2(\mathbb{R})$?

   (c) What conclusion can we make regarding the point of intersection of two lines in $P_2(\mathbb{R})$ whose corresponding lines in $\mathbb{R}^2$ are parallel?