PMAT 3331 – Projective Geometry Assignment #3
Winter 2003

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 3:00 pm on February 12, 2003.
- Please place your completed assignment in Box 35.

1. Suppose we have a set of points $\mathcal{P}$ and a set of lines $\mathcal{L}$ that together form an affine plane. Let d be a dilatation for this affine plane, such that d is not the identity, and such that d fixes the point C. Prove that every line that is fixed by d passes through C.

2. Let $\mathcal{P} = \{1, 2, \ldots, 9\}$, and let $\mathcal{L} = \{\{1, 2, 3\}, \{4, 5, 6\}, \{7, 8, 9\}, \{1, 4, 7\}, \{2, 5, 8\}, \{3, 6, 9\}, \{1, 5, 9\}, \{2, 6, 7\}, \{3, 4, 8\}, \{3, 5, 7\}, \{2, 4, 9\}, \{1, 6, 8\}\}$ be a set of lines on $\mathcal{P}$.

   Is there a dilatation $d : \mathcal{P} \to \mathcal{P}$ such that d fixes point 1, but d is not the identity? If yes, find such a dilatation d. Otherwise prove that there is no such dilatation.

3. Let $\mathcal{D}_C$ be the set of all dilatations that fix a point C in an affine plane. For two dilatations $d_1, d_2 \in \mathcal{D}_C$, define their composition $d_1 \circ d_2$ to be the dilatation that results from applying d_2 followed by d_1, so $(d_1 \circ d_2)(X) = d_1(d_2(X))$ for each point X in the affine plane. Prove that $\circ$ is an associative operation on $\mathcal{D}_C$.