PMAT 3331 – Projective Geometry Assignment #2
Winter 2003

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 3:00 pm on February 5, 2003.
- Please place your completed assignment in Box 35.

1. Which of the following sets of vectors are bases for $A_3(\mathbb{R})$?

   (a) $\{(0,-3,2),(1,9,3),(1,3,7)\}$

   (b) $\{(2,1,0),(0,4,-1),(1,1,0)\}$

2. Let $(K,A)=A_n(\mathbb{R})$. Theorem 1.4 asserts that $(L(A),+)$ is an abelian group, i.e.:

   (a) $L(A)$ is closed under $+$

   (b) $+$ is associative

   (c) $\exists i \in L(A)$ such that $f+i=f=i+f$ for each $f \in L(A)$

   (d) $\forall f \in L(A)$, $\exists(-f) \in L(A)$ such that $f+(-f)=i=(-f)+f$

   (e) $+$ is commutative

   Prove statement (c) above.

3. Let f be a linear form over $(K,A)=A_n(\mathbb{R})$ defined by $(c_1,c_2,\ldots,c_n) \in \mathbb{R}^n$ so that $\vec{a}f = (x_1,x_2,\ldots,x_n)f = \sum_{j=1}^{n} x_j c_j$. Then what is $(-f)$, the additive inverse of f in the group $(L(A),+)$?

4. Find a linear form f over $(K,A)=A_2(\mathbb{R})$ such that $\vec{b_i}f=k_i$, where $\vec{b_1}=(3,4)$, $\vec{b_2}=(4,5)$, $k_1=-2$, $k_2=3$.

5. Find a linear form f over $(K,A)=A_3(\mathbb{R})$ such that $\vec{b_i}f=k_i$, where $\vec{b_1}=(1,0,2)$, $\vec{b_2}=(0,1,3)$, $\vec{b_3}=(1,1,0)$, $k_1=6$, $k_2=3$, $k_3=0$.

6. Find a linear form f over $(K,A)=A_3(\mathbb{R})$ such that $\vec{b_i}f=k_i$, where $\vec{b_1}=(1,0,0)$, $\vec{b_2}=(0,1,0)$, $\vec{b_3}=(0,0,1)$ and k_1,k_2,k_3 are constants.

7. Let $\mathcal{B}=\{\vec{b_1},\vec{b_2},\vec{b_3}\}=\{(1,2,1),(0,1,1),(1,1,3)\}$ be a basis for $(K,A)=A_3(\mathbb{R})$. Find a basis $\mathcal{G}=\{g_1,g_2,g_3\}$ for $(K,L(A))$ such that $\left(\sum_{j=1}^{3} x_j\vec{b_j}\right) g_i = x_i$ for each $i \in \{1,2,3\}$.