PMAT 2320 – Discrete Mathematics Assignment #1
Winter 2008

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at

The following symbols will be used to represent certain sets of numbers:

$\mathbb{Z}$	the set of integers
$\mathbb{N}$	the set of natural numbers, namely $\{1, 2, 3, \ldots\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{Q}$	the set of rational numbers

1. Determine whether the following statements are true, false, or invalid.
   (a) If $4 \geq 4$ and 7 is odd, then $x^2 + 9x - 1$ has a real solution.
   (b) Suppose n is a non-negative integer.
   (c) If $-2^2 = 4$ then 3 is even and $5 < 4$.
   (d) $\sqrt{x^2} = x$.
   (e) 0 is positive.
2. For each valid statement in Question 1 that is an implication,
   (a) state the converse
   (b) determine whether the converse holds.
3. What is the negation of each of the following statements:
   (a) A or (B and not(C))
   (b) A or B or C
   (c) (A and not(B)) and (C or not(D))

Definition. For integers a and b, we say that a divides b (written as "$a|b$") if there exists an integer n such that $b = na$. An integer x is said to be even if $x = 2k$ for some integer k. And an integer x is said to be odd if $x = 2k + 1$ for some integer k.

4. Show that the following statements are false:
   (a) Given that $n \in \mathbb{N}$, $8|n^2$ if and only if $8|n$.
   (b) If $x, y \in \mathbb{R}$ such that $x > 0$, $y > 0$, then $(x + 3)^2 + (y + 4)^2 \leq 5^2$.
   (c) For all $x \in \mathbb{R}$, $100x^4 > \dfrac{x^6}{1000}$.

5. Rewrite the following statements as English sentences. Also indicate whether each statement is true or false.

   (a) $\exists x \in \mathbb{Z}, \exists y \in \mathbb{Q},\ x = y$.

   (b) $\forall x \in \mathbb{Z}, \forall y \in \mathbb{Q},\ x = y$.

   (c) $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Q},\ x = y$.

   (d) $\exists x \in \mathbb{Z}, \forall y \in \mathbb{Q},\ x = y$.

6. Find the negation of each statement in Question 5 and indicate whether it is true or false.

7. Let $x, y \in \mathbb{Z}$. Prove that xy is odd if and only if x and y are both odd.

8. Consider the statement: $\forall x \in \mathbb{Z}$, x is odd $\Rightarrow 4|(x^3 - x)$.

   (a) Is this statement true or false? Justify your answer either with a proof or else with a counter-example.

   (b) What would be the negation of the statement?