

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 17:00 on Thursday October 6th in Assignment Box #23.

1. Determine whether the relation \mathcal{R} is reflexive:

(a) $\mathcal{R} = \{(x, y) \in \mathbb{Z}^2 \mid x^2 + y^2 \text{ is odd}\}$

(b) $\mathcal{R} = \{(x, y) \in \mathbb{Q}^2 \mid xy \geq 0\}$

2. Determine whether the relation \mathcal{R} is symmetric:

(a) $\mathcal{R} = \{(x, y) \in \mathbb{N}^2 \mid x + y = 10\}$

(b) $\mathcal{R} = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^4 \geq 10\}$

3. Determine whether the relation \mathcal{R} is antisymmetric:

(a) $\mathcal{R} = \{(x, y) \in \mathbb{R}^2 \mid x \geq y\}$

(b) $\mathcal{R} = \{(x, y) \in \mathbb{R}^2 \mid x^2 \geq y^2\}$

4. Determine whether the relation \mathcal{R} is transitive:

(a) $\mathcal{R} = \{(x, y) \in \mathbb{N}^2 \mid x + y = 10\}$

(b) $\mathcal{R} = \{(x, y) \in \mathbb{Q}^2 \mid x + y \in \mathbb{Z}\}$

5. Define the relation \sim on \mathbb{Z} by $a \sim b$ if $3a - b^2$ is even.

(a) Prove that \sim is an equivalence relation.

(b) What is $\bar{4}$?

(c) What is \mathbb{Z}/\sim ?

6. Define the relation \sim on \mathbb{R}^2 by $(a, b) \sim (c, d)$ if $a + b = c + d$.

(a) Prove that \sim is an equivalence relation.

(b) Provide a geometrical description of $\overline{(2, 2)}$.